WHAT CAN BE LEARNED FROM THE PROCESS AND RESULTS OF MODELLING THERMAL MASS IN DWELLINGS?

by

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- Bob, for challenging me to go beyond what I thought I could do. It seemed to work.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

This thesis documents an attempt to learn about the impact on energy use of thermal mass in dwellings through the creation and evolution of a purpose-built model. Using insight and techniques obtained from the literature, an increasingly complex model is gradually developed, experiments being carried out at each stage to take advantage of particular properties of each phase of the model before they are obscured with the added complexity of the next phase. As the model grows, it is shown that there is a trade-off between two types of usefulness: inclusion of more of the 'real world' against transparency and ability to solve quickly and/or analytically. The final model, programmed in Matlab, calculates overall annual heating and cooling energy use over varying thermal mass thickness and varying value of one other chosen input parameter at a time.

Main results are that occupancy schedule and heat loss characteristics are the most important parameters of those included in the model, followed by building volume and internal gains. Additionally, room contents contribute significantly to the thermal mass of a lightweight construction. The result of this is the shifting of the dwelling's thermal mass into the most sensitive region of the model space, causing its energy use to be less predictable. In this way, the energy performance of thermally heavyweight buildings is more predictable than that of lightweight buildings.

Following presentation of the results, much discussion is undertaken. Topics include the insight that can be gained given the limitations of the model, and also how to communicate this insight from the realm of mathematics to those who have to make practical decisions on thermal mass use.

1. INTRODUCTION

1.1. Thermal mass: a difficult choice to make

When one asks whether thermal mass in buildings is beneficial in terms of reducing energy use, one is usually told, “It depends”. This is true, but not altogether helpful. “It depends” is a short way of saying that there are many variables and interactions between variables involved, which means that for a given building and set of conditions, it is difficult to know whether thermal mass will be a help or a hindrance.

In this thesis, UK dwellings are used as the context, and the energy-saving properties of thermal mass are examined in a series of increasingly complex purpose-built models.

One thing is certain: it is evident why people are confused. In real buildings, variables associated with thermal mass interact and produce a non-linear energy function. Even for a simple model of the real system in which input variables do not interact, the relationship between energy use and the input variables is not trivial. This thesis seeks to find out what we can learn by this more simple treatment of the system. Throughout, a healthily cautious attitude to the meaningfulness of the model outputs is taken, but it is decided that some insight can be gained. However, when we think we have learned something useful, can we communicate it? Amongst the complexity of the model space, are there pockets of simplicity, intuitive relationships, outcomes which can be passed on as general design principles?

1.2. Aims

- To abstract what the essence of a dwelling thermal mass system is;
- To develop one or more models of the behaviour of thermal mass in dwellings, drawing out lessons about modelling as we go along;
- To explore the model space and learn about what causes switching of the optimum between thermally lightweight and heavyweight construction;

- To reflect on what insight models such as the one developed here can give, to a variety of types of person.

1.3. Chapter-by-chapter overview of the thesis

Chapter 2 reviews literature over a wide range of aspects of the problem. Its main categories are building simulation and thermal mass, to equip us with methods and good practice, cautionary points to be aware of, tests for validity, and more, for use in the rest of the study.

In chapter 3 the process of creating a thermal mass model begins. This chapter documents the model's development from a basic steady state equation to a simple analytic differential equation. On the way, experiments such as the effect of room contents on effective thermal mass are carried out, and the drawback of thermal mass in situations of intermittent occupancy is demonstrated.

Chapter 4 continues the model development process, discussing which are the important effects to include. Once selected, these effects are modelled using mathematical functions and the final differential equation constructed and presented.

This equation is validated in chapter 5. It can now start being used for experiments. The first one involves demonstration of the beneficial side of thermal mass (compared to the negative side shown in chapter 3) – its ability to smooth temperature. The chapter goes on to define a general objective function, which takes both characteristics into account, as overall annual energy use. A program is written to investigate this over a range of thermal masses for different input parameters.

In chapter 6, results of the final model are presented in graphical form, although there is a discussion of the difficulty of presenting the model space. This chapter attempts to discover which input variables are the most important

(in general, and in certain situations). Assumptions of the model are clearly stated.

Chapter 7 is meta-analysis of what really can be learned, given the assumptions just presented and the philosophy of modelling section in chapter 2. This chapter is addressed to three audiences over a variety of disciplines.

Chapter 8 summarises the study, gives suggestions for further work, and gives some concluding remarks.

2. LITERATURE REVIEW

Since this thesis concerns not just the physical behaviour of thermal mass but also the process of modelling it, the literature review should cover both territories. Thus, it will start with a general section on building simulation to lay down some principles on what can make a good model, before going on to discuss thermal mass. Finally, previous efforts to link the two are critiqued.

2.1. Building simulation

Philosophy of building simulation

Literature linking building simulation to the general philosophy of modelling does not seem to exist, so the author will infer that building simulation is a type of "model-based science" – a term used by Godfrey Smith, 2006 and others before. With this categorisation comes application to the models in this thesis of aspects of model-based science: its definition, its expectation of verification and validation, and its warnings.

i) Definition: what should be created

"*What is most distinctive of model based science is a strategy of indirect representation of the world.[...] The modeller's strategy is to gain understanding of a complex real-world system via an understanding of simpler, hypothetical system that resembles it in relevant respects."* (Godfrey-Smith, 2006)

The kind of building simulation which will help understand the role of thermal mass should indeed reduce the system down to its most important elements, or its "essence" (Gleik, 1988). A fairly unsuccessful effort to determine from the literature what this essence is in the case of thermal mass modelling is undertaken on page 26.

ii) Verification and validation: what should be tested

A widely-used principle is that,

"*...validation of a simulation is the process of assuring that the model equations that are the basis for the simulation represent the target system correctly. Verification, on the other hand, is the process of assuring that the numerical output of the simulation, and the conclusions drawn from them, are close enough to what the solutions of the original model equations would be if we were able to write them down.*" (Winsberg, 2009)

This quote talks of ensuring firstly that the model's underlying equations are suitable, and secondly that if a program is used, an appropriate solver is invoked.

Unfortunately Winsberg, having acknowledged this classification, goes on to challenge it – however, it is good enough for this thesis. Chapter 5 will carry out some validation tests; verification will happen as iterations of the model are presented in subsequent chapters.

iii) Warnings: what should be kept in mind

"*When we calculate things within [a] model, we learn, in the first instance, about the model world*." (Morrison & Morgan, 2001.)

This is a useful point to have in our minds as the model in this thesis is developed and results are obtained: they are results of the world described by the model. Inspired by a talk by Shipworth (2010) referencing Godfrey Smith (2006) and Giere (1988), the author considers the transition from the real world, to the world inside the modeller's head, to what the modeller actually programs and what, here, the software outputs, as a series of dilution and pollution of real world effects.

All this section on model-based science serves not just to cast doubt on the results obtained in this thesis, but to ensure they are recognised as what they really are.

Optimization in building simulation – why, how, whom

Since this thesis seeks to gain insight into the behaviour of thermal mass such that its use in dwellings can be optimized, it is worth learning how people are starting to use optimization techniques in building simulation, and evaluating what out of their methodology is appropriate to use here.

An optimization problem consists of (Wetter, 2002):

- A set of free parameters (often in this thesis called input variables);
- Some constraints which bound the domain of the input and dependent variables;
- An objective function, which is the function to minimise, and which depends on the independent variables.

In industry, a design will typically be passed to a modeller to test its energy performance in software such as TAS or EnergyPlus. The modeller may spend one or two days running perhaps between one and five simulations, changing a few parameters and trying to improve the energy performance.

Recently, optimization algorithms have been linked to or integrated within building simulation packages. An example of the former is Genopt and of the latter is BuildOpt (see Wetter, 2010 and 2004 respectively). Such algorithms work in the following way: input of parameters into the model, evaluation of the objective function, feeding of the results back into the optimization module, decision about how to vary the parameters, input of the new set of parameters, and so on. With a careful choice of algorithm, the main features of the model space can be explored and the global optimum found by carrying out a factor of at least ten less evaluations of the objective function (see e.g. Holst, 2003). The large number of simulations carried out in this thesis (see chapter 6) could potentially be reduced by the use of such an optimizing algorithm.

However, there are three reasons why the use of an optimization algorithm is not appropriate in this thesis: one for us, and two for people who build houses. The first is that the presentation of a single result as the optimum design of a

building teaches us practically nothing – as opposed to exploring the model space (to be defined in chapter 6) ourselves. Secondly, designers rarely use one criterion in actual building design. For example, Coley & Schukat (2002) explain that, "*Because of the difficulty of including such factors as aesthetics in the optimisation process, it is likely that any "optimum" result will be found to be unacceptable*." Thirdly, chapter 6 will show the potential danger of the combination of an algorithm being trapped in a local minimum and an inaccurately-constructed building causing energy use to soar. The second and third of these problematic factors will be brought up again in Appendix 3, which discusses a possible solution in the form of a multi-criterion objective function.

Big models versus small models

How much to include in the model depends on the nature of the desired solution. If an optimization algorithm is to be used, big models can be acceptable as evaluation of the objective function, which can take hours, only has to be carried out a few times. However, to try to really understand an effect, the author feels that a small model – i.e. one which does not take long to run, which captures only the most important effects and in which all input functions and variables are known – can provide more insight than a big model such as EnergyPlus. Other fields, such as system dynamics, have been moving recently towards small models "to gain insight", using the argument that they "align better with mental models" (Forrester, 2006).

As an aside, if the model is ever to be used as a 'tool' for others to play around with (as will be discussed in Chapter 6 and implemented in Appendix 1), big models take so long to run that the user cannot sit around waiting for them.

Over the dissertation, as the model is increased in complexity, the gains and losses in insight this brings will become apparent.

2.2. Thermal mass

Advantages and drawbacks of thermal mass

There are several reasons why a designer would choose to include thermal mass as part of the construction of a dwelling, and several reasons why not. A summary is as follows:

When properly used, it is claimed that thermal mass can reduce and delay peak summer internal temperatures effectively, and enable some passive solar heating in winter. However, it increases the heat capacity of the dwelling, such that more heat is needed to warm it up; if the occupants then leave, that large amount of heat is lost. Another issue is the release of stored heat in summer – if this coincides with a time at which this heat can escape (night ventilation, or a 10K drop between inside and outside (Szokolay, 1984)), the occupants never have to be exposed to that heat – but if neither of the above conditions is satisfied, conditions in the building can be unpleasantly hot.

The reader may have noted that very little of that summary was backed up by reference to literature. This is because much of the literature exhorting the benefits of thermal mass is authored or co-authored by the concrete industry, and therefore one has reason to be hesitant before treating it as ‘objective’...

For example, a recent study carried out by Arup in 2007 showed that over the lifetime of a house (and including the predicted effects of climate change) the use of thermal mass can reduce the dwelling’s CO_2 emissions by up to 17%. The study also happened to be co-authored by the British Cement Association and the Concrete Centre (Hacker et al., 2007). The American equivalent, a report for the National Association of Home Builders in 1999, showing 20% savings in energy by the use of thermal mass, was funded by the Portland Cement Association and Reddi-Form Concrete (NAHB Research Center, 1999).

Much of the literature not supported by the concrete industry testing empirically the performance of thermal mass was carried out in the 1980s (in America) using test cells. It was problematic, as documented by Lomas &

Bowman (1987). For example, a series of tests carried out at Los Alamos had several less-than-ideal approximations: false air infiltration, not enough measurements taken to deduce the heat balance in the cells, and seemingly no lightweight or control cell.

In the UK, the Building Research Establishment (BRE) tested the 'Hanson EcoHouse' in its Watford site for two years and "demonstrated the benefits of high levels of thermal mass [...]" (BRE, 2010). Although the house was funded essentially by the cement industry, it is claimed that it was actually BRE who carried out the tests on it. "[The tests] proved the house's ability to stay cooler in summer and warmer for longer in winter." However, how the tests were done and whether intermittency was accounted for is not public information.

The 'reputation' of thermal mass

Thermal mass has two types of reputation amongst the environmentally-interested public: good, and confused. For example, a quick peruse of websites offering advice on energy saving in the home (again excluding the numerous offerings of the concrete industry) yielded optimistic comments such as:

"Eco-buildings are usually designed to have a high thermal mass for several reasons..."(Marshall, 2002).

Also, there are many signs of confusion and people being told wrong information. Looking at one green building forum (AECB Forum, 2006) yields the following:

"There is a lot of nonsense spoken about thermal mass keeping the heat in."

"At the end of the thermal day it's a complex interaction between thermal gains, insulation, thermal mass, admittance etc - employ an energy consultant!"

A thought provoked by writing this thesis is the need to clearly explain to non-specialists in what situations thermal mass saves/wastes energy, and perhaps to enable them to discover this themselves.

Modelling thermal mass: who has done it, and what parts of the system are important?

i. Thermal mass models

Many thermal mass models are overviewed in Balaras (1996). The following are picked out from his paper because of their interesting methodology, focus or results.

A ‘large’ model is considered first. The meaning of ‘large’ here refers to those models containing up to 200 input variables! (Element Energy, 2010.) One such study linking optimization techniques with thermal mass use (Braun, 1990) was actually devised to find ways to reduce the *cost* of energy services. It is worth including here because the method illustrates a problematic aspect of such models: some input variables were not continuous for example, wall constructions. Therefore, the objective function was not smooth, and this required a certain type of optimization algorithm. The result was that the computing requirements to carry out the optimization were enormous.

This warns us against using a very complicated objective function: if it takes a computer days to solve, there is no way that the result can be checked in the modeller’s head and there is not much to be learned from it really, especially given what we learned about the benefits of small models on page 8.

Findings from this paper included that both energy costs and peak electrical use can be significantly reduced through proper control of the building's thermal storage. The authors also discussed what the model was sensitive to - see ii) below.

An interesting second problem treated in this paper involved minimizing peak electrical power demand. Since power, as opposed to overall energy use,

might be the concern of the future (Oreszczyn, 2010), this is an important issue to consider, albeit beyond the scope of this thesis.

The authors of the next study to be discussed (Givoni & Hoffman, 1973) worked in terms of the 'thermal time constant of the building' for measuring the ability of the enclosure's interior mass to admit and store heat. Their metric will be discussed and included in chapter 3, as it is an intuitive way of comparing different effects: for example, how much longer does a building take to cool down when it has 'clutter' in it (see chapter 3). Another aspect of their study worth mentioning is that it was validated using real data. This thesis does not have numerical validation, just a kind of visual validation (see chapter 5), since it does not model a real dwelling but a hypothetical one.

As the paper's result is prediction of internal temperature, it does not contain the type of results that would be useful here; it is more the metric used that is interesting for this thesis.

Mathews and Richards (1991) use a methodology that seems feasible for this thesis. Input variables are treated as periodic, a single zone at uniform temperature is assumed, and hourly air temperature and sensible energy loads are predicted with the effects of heat storage. These authors' claim of validity was as follows:

"*The model has been extensively verified in more than 60 passive, naturally ventilated buildings. For 80% of the time temperature predictions were within 2°C of measurements, showing that heat storage effects are efficiently simulated*."

This is an amazing claim to make, given the rather blanket input assumptions outlined above. This paper struck the author as containing an interesting way of looking at the system to be modelled: as a thermal network. This idea will be fully explained in chapter 4.

Unfortunately, the author cannot trace what the model was used for afterwards, and if it was used, the results obtained.

In Peacock et al. (2010), three buildings of different thermal masses were simulated using a software package called ESP-r. The simulations suggested that low thermal mass dwellings will be prone to overheating as the UK climate changes:

"*A significant proportion of the existing UK housing stock can be considered to be low thermal mass and will therefore be disadvantaged by extant climate change. However, this is not to suggest that the existence of future overheating will be solely determined by the thermal mass of a dwelling. With an optimistic window-opening adaptation strategy, the advantages of increased thermal mass when compared to the lighter-weight variants investigated become less pronounced*." (Section 5, Peacock et al. [2010])

This paper, however, treats the independent variable of quantity of thermal mass by presenting discrete scenarios (e.g. high thermal mass versus low thermal mass), not a continuous spectrum (which would enable the reader to get a feeling for any trends). This thesis will differ in this respect.

ii. Important factors of the thermal mass problem

There does not seem to be a consensus as to which are the most important or sensitive factors that necessitate accuracy (sometimes called 'critical performance parameters, see for example Orme et al, 2003) when one is calculating potential energy savings from thermal mass. Various opinions can be found in the literature:

- Weather conditions: Peacock et al. (2010) showed (by simulations) that an important factor is which UK climate file is used. Weather was also acknowledged as important by Braun (1990) and Szkolkay (1984). Peacock et al. make the interesting comment that perhaps buildings should be built differently in different parts of the UK – a point which will be returned to in chapter 7.
- Location of the mass: Orientation and location of thermal mass within the building can drastically influence its effectiveness (Balaras, 1996).

- Properties of the mass: High thermal diffusivity enhances effectiveness (Braun, 1990).
- Convective losses: Ventilated air enhances convective heat losses from mass (Balaras, 1996).
- Occupant behaviour: occupants opening windows cause loss of internal gains that could contribute to heating when external temperatures fall (Willoughby, 2002).
- Part-load characteristics of the cooling plant and air handling system (Braun, 1990): this will not be addressed at all in this thesis since it is fairly complex but is probably quite a significant issue.
- Metric/objective function: Peacock et al. (2010) pointed out that using different metrics to measure effects causes different results. One example is the choice of overheating metric: if average daytime internal temperature is used as opposed to that at night, the reduction in energy use from using higher thermal mass is halved. This will be a good lesson to keep in mind in throughout this thesis: one (of many) reasons not to take the results too literally is that only one metric is used, and comparison with another could potentially change the results a lot.

The literature offers a range of potentially influential factors, no source showing numerically which is most important. The most rigorous study appears to be Mathews and Richards (1991), whose methodology will be drawn from in subsequent chapters.

2.3. Conclusion to literature review

After visiting many topics, the following paragraph summarises what lessons can be carried forward to use in the forthcoming model's development and employment:

A model of thermal mass in a dwelling will be created. It should be a relatively small model, such that it can be aligned with a mental model. An advantage of this model compared to others will be the treatment of thermal mass as a

continuous variable as opposed to an assortment of cases. An optimization algorithm could be used but perhaps will not be helpful. The results output by the model should not be taken too literally, for the following reasons: it is the model world not the real world being examined, only one results metric is considered, there is not much empirical data to test it against (especially for the UK), and even though we shall try to capture the ‘essence’ of the system, there is no consensus on what that is.

3. DEVELOPMENT OF THE EQUATION - 1: STEADY STATE TO ANALYTIC O.D.E

In this section, an equation underlying a thermal mass model will evolve from its simplest possible form to slightly more realistic.

In order that we can start somewhere easy, we will define an initial objective function of heating energy used. There are several types of equation could be used to calculate heating energy. The following section will start with a very basic equation, see what can be learned, and incorporate more features to make the equation a bit more realistic.

3.1. The steady state equation

As a warm-up, a steady state equation of heat loss is written down. It does not even include thermal mass – it simply illustrates heat loss and points out a few noteworthy aspects of the problem.

$$Q = H(T - T_o) \qquad \text{Equation 1}$$

Q = heat loss, W
H = heat loss coefficient, W/K
T = internal temperature, °C
T_o= external temperature, °C

Let us now use the following well-known approximation to model heat loss:

$$H = \sum UA + \frac{1}{3}nV \qquad \text{Equation 2}$$

U = thermal transmittance, W/m^2K
A = area of surface subject to heat loss, m^2
n = hourly air change rate, h^{-1}
V = volume of dwelling

Equation 2 models heat loss through conduction and convection through its two terms respectively.

Let us now make an observation about H by considering a small, box-shaped dwelling – more like a room – of 5m x 5m x 2.5m.

Inserting some typical values into Equation 2, it can be seen that the convection term, $\sum UA$, is more important here than the conduction term, $\sum \frac{1}{3} nV$. Table 1 shows this effect.

Table 1: Comparing the conduction and convection terms in the heat loss coefficient.

U (W/m^2K)	A (m^2)	conduction term (W/K)	n (ach^{-1})	V (m^3)	convection term (W/K)
0.1	50	5	0.5	62.5	31.3
0.2	50	10	1	62.5	62.5
0.5	50	25	2	62.5	125

This is probably the smallest possible likely dwelling, and therefore the lowest volume/area ratio. It is worth noting that as the model dwelling gets larger, V/A increases, so the convection term of Equation 2 becomes increasingly more important than the conduction term. Thus, assumptions and simplifications made about convective losses become increasingly important.

Equation 1, since it does not include thermal mass, is too simple to use beyond this point. Since the effects of thermal mass are demonstrated in time, the equation chosen should not be steady-state, but dynamic.

There are two types of dynamic equation: differential equations, and difference equations, In terms of dynamic equations the more that can be done through analytic equations not requiring numerical calculations, the better. One reason for this is firstly that equations can be solved analytically, and therefore rearranged to get any desired variable/parameter (an example of this is shown in the next section). Therefore differential equations will be pursued next.

3.2. First differential equation

A single-zone heat-balance model including thermal mass would look something like:

Heat gains per unit time = heat storage per unit time + heat losses per unit time.

$$G = C\frac{dT}{dt} + H(T - To)$$

Equation 3

G = heat gains (both through free gains and heating systems).
C = thermal mass of everything in that zone, i.e. walls, air, contents of the room.
T_o = outside temperature; here assumed constant.

To give an example of the type of insight which can be gained from this simple differential equation, a problem in building physics will be used. Please see Figure 1:

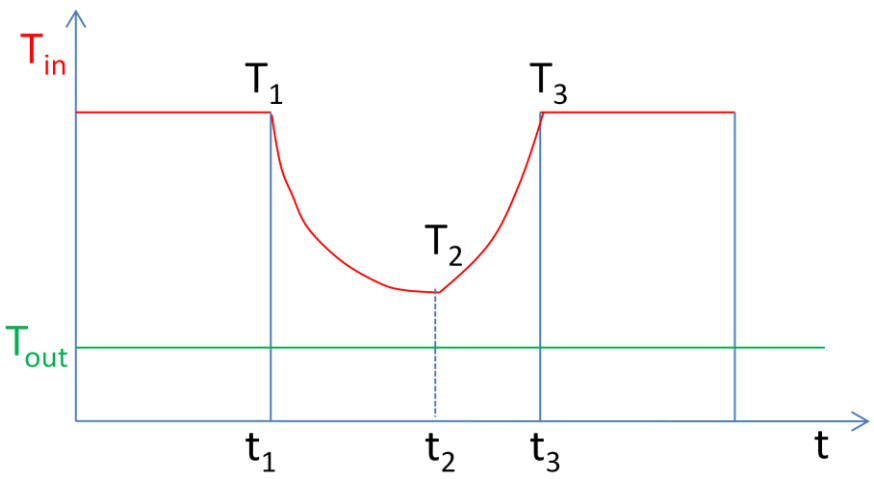

Figure 1: heating of an intermittently-occupied space.

Figure 1illustrates 'intermittency': occupants of a building going out during the day. This diagram is explained below:

- In winter, a thermostat controls the temperature such that at times when the occupants are in, it is at temperature T_1.

- They leave the house at time t_1, so the heating is switched off,
- The heating comes back on at time t_2 (which is not determined yet)…
- ...such that the house is at temperature T_1 by the time the occupants return at time t_3.

Here, the heating system has fixed power, so the earlier it has to come back on (i.e. the lower t_2 has to be), the more energy is used.

To minimise overall heating energy use in this situation, should one have a house with high thermal mass which will not cool down as much but requires more energy for each degree it has to heat back up, or a lightweight house which cools more, but is easier to heat back up? In other words, which type of house needs the heating to come back on earlier?

This problem is expressed mathematically in Box 1. The horrified reader can skip straight to the solution on page 21.

Box 1: Mathematical solution of preheating time for different thermal masses.

The point t_2, which is the intersection of the cooling curve and the heating curve in Figure 1, must be found.

From Equation 3,

Cooling curve:

$$0 = C\frac{dT}{dt} + H(T - To) \qquad \text{Equation 4}$$

Heating curve:

$$G = C\frac{dT}{dt} + H(T - To) \qquad \text{Equation 5}$$

Problem: find t_2.

The nice thing about such a simple set of equations is that they can be integrated analytically, with limits (T_1,t_1) to (T_2, t_2) and (T_2,t_2) to (T_3,t_3) respectively, to obtain:

$$T2 - To = (T1 - To)\exp\left(\left(-\frac{H}{C}\right)(t2 - t1)\right)$$ Equation 6

$$T3 - To - \frac{G}{H} = \left(T2 - To - \frac{G}{H}\right)\exp\left(\left(-\frac{H}{C}\right)(t3 - t2)\right)$$ Equation 7

This pair of simultaneous equations can be solved for t_2 (and T_2 if needed; not shown here):

$$t2 = \frac{C}{H}\ln\left[\frac{e\left(\frac{Ht3}{C}\right)\left(G+H(To-T3)\right)-e\left(\frac{Ht1}{C}\right)H(To-T1)}{G}\right]$$ Equation 8

Examining the analytic solution

Now, since we want to minimise (t_3-t_2) (or maximise t_2 for a fixed t_3) with respect to the amount of thermal mass (C), we can examine the properties of the expression for t_2.

The first thing to do is to simplify it by setting t_1 = 0. This is fine, since t_1 is an arbitrary time when the occupants leave the house. So we are left with:

$$t2 = \frac{C}{H}\ln\left[\frac{e\left(\frac{Ht3}{C}\right)\left(G+H(To-T3)\right)-H(To-T1)}{G}\right]$$ Equation 9

Note that t_3 is now a measure of how long the occupants spend out of the house.

Looking at the mathematical form of Equation 9, we can determine that:

- The most important term is C/H. Now, a term used in the literature (see for example Antonopoulos & Koronaki, 2000) is the 'building time constant', defined as $\tau = \frac{C}{H}$. On inspection, one can see that it is the ratio of effective heat storage and heat loss. It is analogous to the electrical time constant L/R (inductance over resistance) – more comparisons between the dwelling and an electrical circuit will be made in Chapter 4. So the x-axis of Figures 2 and 3 can be thought of as a building-size-independent measure of how well heat is stored as opposed to lost.
- Equation 9 is differentiable with respect to C. We could use this to

determine whether there are stationary points at which t_2 is greatest at a given thermal mass, but the derivative is mathematically horrendous, so is not presented here.

- Instead we will investigate what happens to t_2 at the limits of small and large C/H, with all the other parameters fixed (intermittency, T_1, etc...):
  - As C/H -> 0, t_2 -> t_3, i.e. for lightweight buildings, we expect to not have to have the preheating on for long.
  - As C/H -> ∞, t_2->0*, so preheating time is longest.

*except in one special case which is mathematically possible but in reality unlikely: there is a stationary point in Equation 9 when $T_3 < T_1$.

Figures 2 and 3 illustrate the behaviour of preheating time for different ranges of C/H, to make the point that lightweight buildings always come out as favourable wherever one goes exploring on the C/H axis:

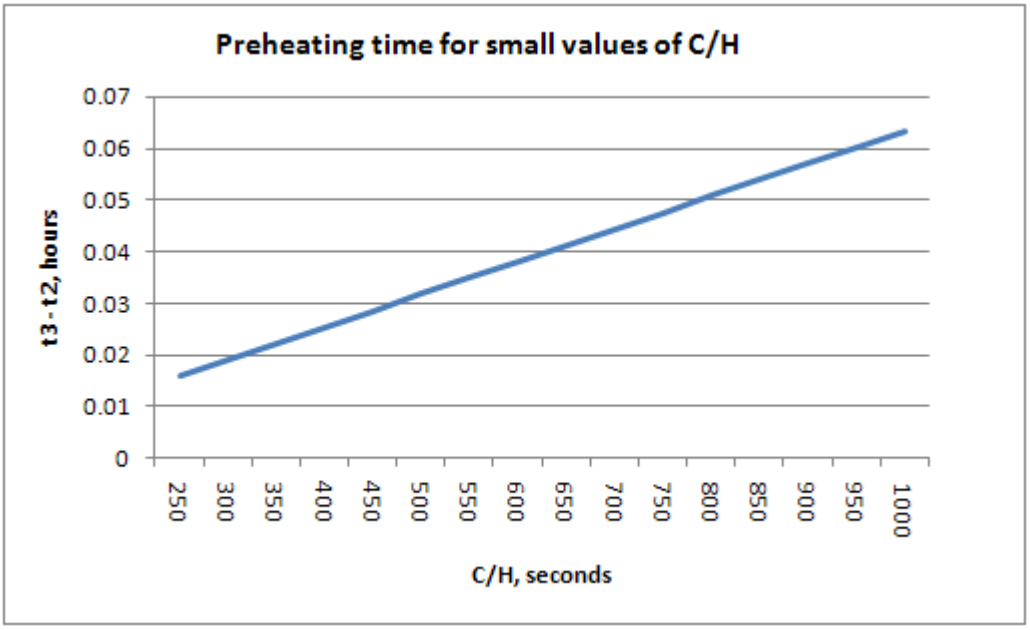

Figure 2: Preheating hours for small building time constant.

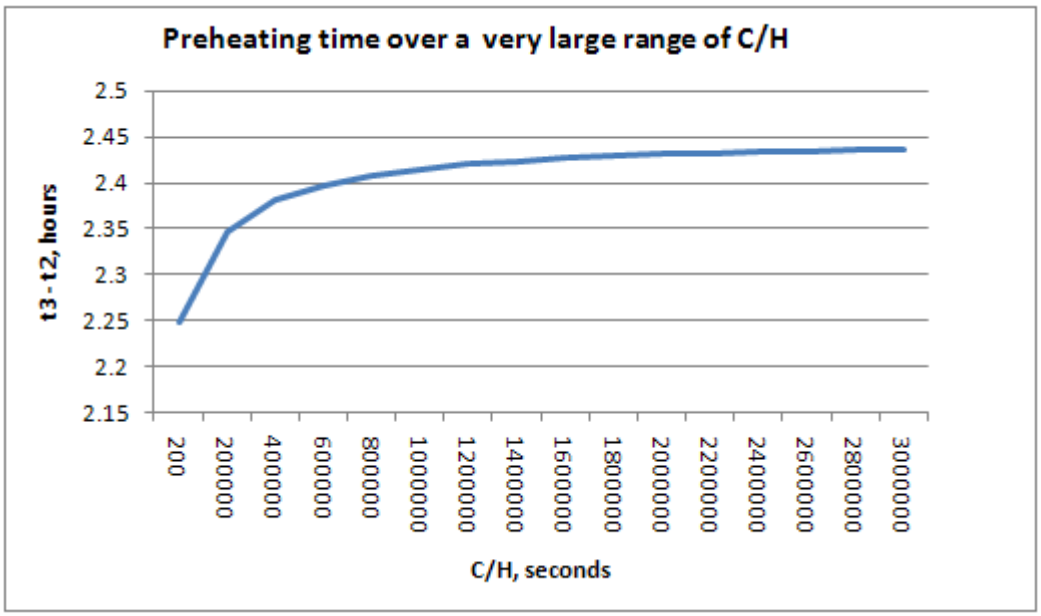

Figure 3: Preheating hours for large range of building time constant. Note that the first point is not quite to scale (the equation blows up if 0 is used as the independent variable).

Another way of plotting Equation 9 is to fix thermal mass into 2 cases, 'lightweight' and 'heavyweight', and vary intermittency, to see the effect on hours of preheating time:

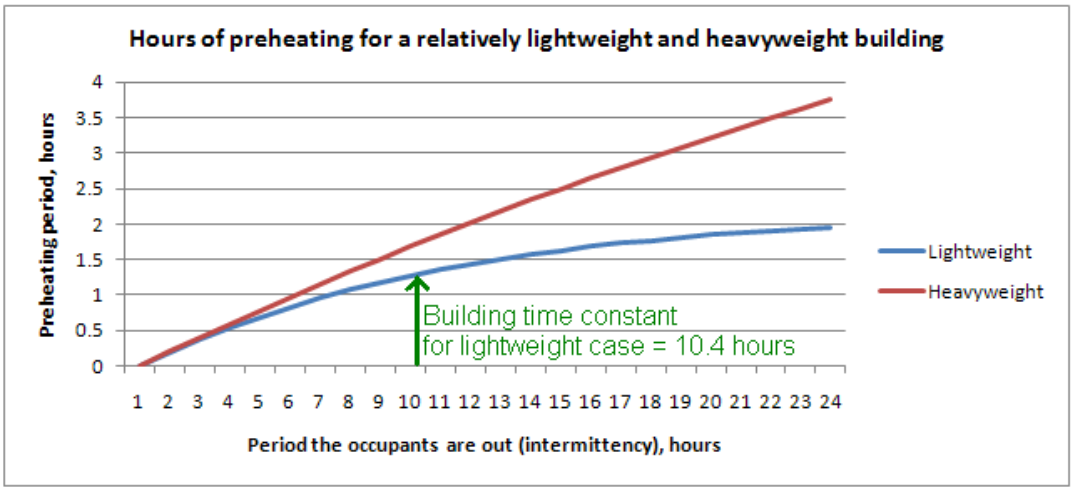

Figure 4: Comparison of preheating times over a range of intermittencies, for different thermal masses

Table 2: Assumptions going into Figure 4

Type of construction	Thermal mass, J/K	Assumption to create this value of thermal mass
"lightweight"	700,000	Walls covered in plasterboard
"heavyweight"	3,000,000	Walls covered in 100mm exposed concrete

Again, it can be seen that preheating time is longer for the heavyweight building, the effect being magnified as intermittency increases.

Learning from simultaneous equations: the drawbacks of thermal mass

This simple examination of an analytic equation has shown us that, whatever the intermittency, thermal mass is always a hindrance to preheating energy if the external temperature is always below the desired internal temperature.

Of course, this is just one characteristic of thermal mass and one piece of the problem, but in chapter 5 when we return to this characteristic in a non-analytic case, and when the solution to the equation becomes non-intuitive, having carried out this analytic examination will prove helpful.

3.3. Effect of 'clutter'

While we have this analytic equation, it is good to get the most out of it. Another discussion we could have is the effect of 'clutter' – objects in the space which, although not designed to be thermal mass, act as such. The extent to which clutter imitates thermal mass and turns a lightweight construction into a heavyweight one will now be examined.

The author conducted a thought experiment to estimate the thermal mass of the contents of the university room in which she was working at the time. Since she unfortunately did not have weighing scales (and even if she did,

would not have trusted herself to lift UCL computers etc), she observed what was in the room, scaled the quantity of that particular object to what it could be in a dwelling, looked up its mass and approximate thermal mass online, and multiplied them together. The results are presented in Table 3. The author notes that they are subject to the internet and not her own measurements and should be taken as a rough guide only.

Table 3: Estimated thermal mass of 'clutter' in the space

Object	Likely quantity in the dwelling space	Material	Mass per object, kg	Mass per m^2, kg/m^2	Specific heat capacity, J/(KgK)	Thermal mass, J/K	Reference(s)
computer or other sizeable electronic equipment	2	plastic	45		1670	150300	Engineering toolbox, 2005
desk or table	1	timber	40		2100	96000	Binderholz, unknown
carpet	$25m^2$	carpet		2.5	1000	62500	Vegt, 2005
fittings (lights)	9	metal/glass	2		870	15660	Wilkinson, 2008
Versatemp units (for heating/cooling)	2	metal (assume aluminium)	30		870	52200	Engineering toolbox, 2005
chairs	6	plastic	20		1670	200400	Engineering toolbox, 2005
plasterboard	walls either covered (lightweight)or not (heavyweight).	plasterboard		11	840	693000	Speedline, unknown
					total including plasterboard:	1270060	
					total excluding plasterboard:	577060	

There are three significant results here:

- The thermal mass of a completely empty ‘lightweight’ space of dimensions 5m x 5m x 2m is almost 700,000 J/K, which translates to a cooling time constant of 10 hours, given the default heat loss characteristics assumed throughout (see Table 6).

- When the contents of the room are added, this brings the total to over 1 million J/KgK, which is actually equivalent to that same space being emptied and the walls lined with 40 mm concrete!
- When calculations involving rooms with thermal mass are undertaken, about half a million J/K should be added to the thermal mass to account for room clutter.

These results will be useful later on in chapter 6, when we consider which regions of the thermal mass axis are meaningful.

In reality the clutter is 'fast-response thermal mass', meaning that it re-emits heat quickly, and does not have the time-lag admittance effect of wall thermal mass, but since admittance is not modelled, this distinction does not matter. See Table 8 for the predicted effect of this simplifying assumption on the results throughout this thesis.

3.4. Conclusion to chapter 3: Insight gained in this chapter

Using even a simple analytic differential equation, we have demonstrated one property of thermal mass: its tendency to increase heating energy use when occupancy is intermittent. We have also seen that the equivalent thermal mass of contents of the room not intended to contribute to thermal mass is significant – perhaps equivalent to about 4 cm of intentional thermal mass.

In Chapter 4 we will look at the other side of thermal mass: its ability to smooth internal temperature in a space.

4. DEVELOPMENT OF THE EQUATION – 2: INCLUSION OF MORE EFFECTS

Equation 9, in Chapter 3, was designed to illustrate just part of the behaviour of thermal mass, and is very simple. To further mimic reality, we must choose what else to include. At the end of this chapter, the final equation for the dwelling in free-floating mode will be presented.

Since the literature did not agree on which variables are most important, the author will take as a starting point those included in the most comprehensive paper, Mathews et al. (1991), which was also validated empirically. The relative importance of each input parameter will be questioned and those with a seemingly small influence will be eliminated. We end up with a simplified set of variables, which will form the final equation.

Mathews et al. use the common technique of representing the building as a thermal network. This can be helpful, as differential equations can then be written down just as would be their electrical analogues, the idea of time constants is more natural, and the extent of influence of a component on another one can be seen.

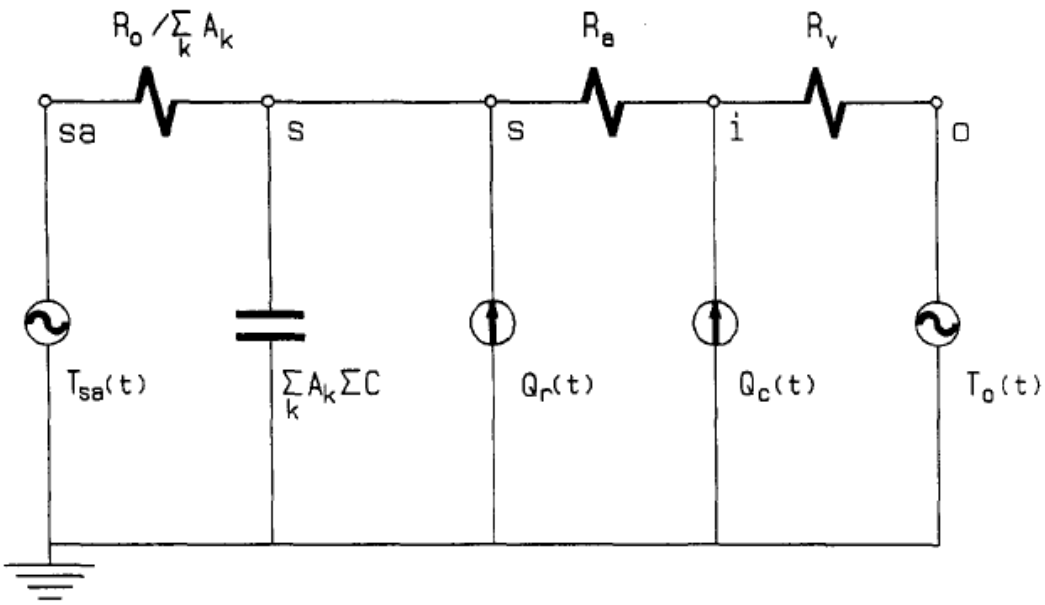

Figure 5: Thermal network showing the building described in Mathews et al, 1991.

Table 4: Meaning of the terms in Figure 5.

Forcing elements	T_{sa} Q_r Q_c T_o	Sol-air temperature. Radiative heat input (into the interior surfaces) Convective heat input (into the air in the space) Outside air temperature
Resistances	$\sum_k \frac{Ro}{A}$ R_a R_v	Heat transfer between the sol-air node and the interior. This includes conduction and the effect of solar gain on the outside of the building. Heat transfer between the indoor air and indoor surfaces Ventilative hear losses, i.e from convection
Energy storage	$\sum_k A \sum C$	Effective thermal storage of the walls.

4.1. What to include

To capture the ‘essence’ of the system as was stated as an aim in Chapters 1 and 2, meaning that which is most important in contributing to the performance of thermal mass, some factors can be eliminated.

Let us try to see the relative sizes of the terms in Table 4, in Watts.

Table 5: Relative sizes of effects contributing to the thermal network in Mathews et al.

Heat phenomenon	Rough calculation method for size off effect, in Watts	What the symbols stand for, if not defined before	Approximate inputs, in this case	Approximate outcome	References
Heat loss through conduction through walls, roof, floor	$UA(T - To)$		Walls: U = 0.2 W/m^{2}K A = 48m^2 T = 20 °C T_o = 10 °C	Walls: ~100W	Equations 1 & 2

Heat phenomenon	Rough calculation method for size off effect, in Watts	What the symbols stand for, if not defined before	Approximate inputs, in this case	Approximate outcome	References
Sensible heat loss/gain through window	$UA(T - To)$		U = 2.85 W/m^2K A = $2m^2$ T = 20 °C T_o = 10 °C	~60W	U-value of double glazing, 12mm spacing: from CIBSE Guide A, Table 3.23 (CIBSE, Guide A: Environmental Design, 2006)
Solar gain through window	$Fc\ x\ Fs\ x\ qsg\ x\ A$	F_c = air-node correction factor F_s = shading factor q_{sg} = solar cooling load	F_c~0.9 F_s ~0.8 q_{sg}: very variable; seems to average around 200 W	~ 300W if it is sunny	q_{sg} approximated from CIBSE guide A, Table 5.19 F_c, F_s taken from online, 2010 (see references)
Convective heat losses/gains to/from exterior	$\frac{1}{3}\ nV(T - To)$		n = 0.5 ach V = 62.5 m^3	~100W	Equation from Chapter 3
Metabolic and equipment heat gains	M =(number of occupants x heat given off by one occupant) E varies according to lifestyle but in a dwelling should not be very significant.		M~100W (estimate one occupant for a dwelling as small as the one being modelled to begin with) E~50W	~150W	M approximated from Cibse Guide A, Table 6.3 (CIBSE, Guide A: Environmental Design, 2006)
External solar gains	Mean gain: $AU(T_{sa}-T)$	T_{sa} = sol-air temperature	Complicated, but assume a value 22°C in summer when T is 21°C.	~10W	online, 2010 (see references) T_{sa}: CIBSE Guide J, Table 5.36 (CIBSE, 2002)
Heat transfer between air and walls	$hA(T-T_w)$	h = convective heat transfer coefficient T_w = wall temperature	h~4 W/m^2K	Not known yet, since the size of the difference between Tin and Tw is not yet determined; experiment to follow	h: Grenfell Davies (2004), page 100
Heat storage in air-wall system	$C\frac{dT}{dt}$			Not known yet, but this term is needed as this is thermal mass, the point of the problem.	Equation 3

Note that latent heat is not even considered in Table 5 as it is not mentioned in the network diagram above, and it is just getting too complicated. But it does flag up the fact that the author has ignored it without really looking into what it is.

It seems that external gains are an order of magnitude smaller than other effects, and sensible window gains are significantly smaller than other terms. These will be left out of the final equation. This does match with other treatments – see arca53 (date unknown) for an example of someone else doing this and getting the same result in terms of relative sizes of effects.

4.2. How many nodes/equations

The thermal network shown in Figure 5 contains multiple nodes, representing heat flow being calculated at multiple points: inside the space (i), at the wall (s), and outside (o). Whether this is a necessary treatment in this thesis or whether the nodes can be condensed into one is a valid question, so its answer was found experimentally. Since this is a fairly long and mathematical treatment, it is left out of the main text and can be found in Appendix 2. In summary, the effect of treating the air and walls separately and hence describing the system in two equations instead of one was not found to make a significant difference, so the thesis will ‘lump’ together (see e.g. Firth, 2010) the walls and the inside of the space into one node, described by one O.D.E.

From Table 5, ‘gains’ large enough to include are: occupant metabolic gains (M(t)) and solar gains (S(t)). Conduction and convection losses should be included, along with the capacitance effect of thermal mass.

The finalised network can be shown as follows:

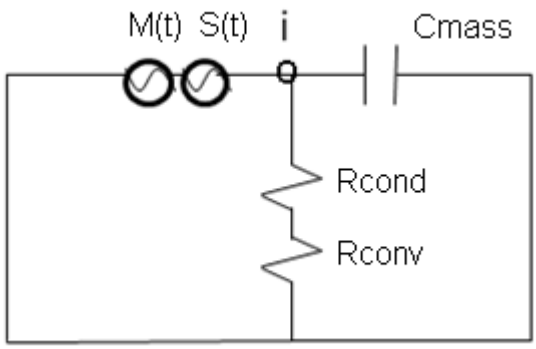

Figure 6: Final thermal network

The ‘capacitor’, which is the thermal mass, has a time constant of C/H, which is the analogue of the electrical time constant L/R – a measure of energy stored compared to energy lost from the system.

The resistances, R_{cond} (conduction losses) and R_{conv} (convection losses) both act on the same node (a combined wall-air node), and the forcing elements all act on this node too.

From Kirchoff’s law (the sum of currents flowing into a node equals the sum of currents flowing out) we can write down the free-floating equation for the node:

$$M(t) + S(t) = C\frac{dT}{dt} + H(T - To(t))$$ Equation 10

4.3. Making functions

The next task is to input M(t), S(t) and To(t) – which are metabolic and equipment gains, solar gains and outside temperature respectively. Either empirical data could be used, or mathematical functions. The latter alternative is chosen as then noise is eliminated and the inputs do not do anything unpredictable. This does however further remove the model from the real world. A small incorporation of reality is from the numerical constants in the functions made all being based on real data: TAS weather file for London Heathrow (Test Reference Year).

Metabolic rate

The mathematical nature of the inputs unfortunately does not lead to the resulting equation being solvable analytically. What kills the analytic solution is metabolic rate.

It was assumed to begin with that there is one occupant, and for the moment he/she is only in for 12 hours at a time – between 6pm and 6 a.m. (This is modified in Chapter 6).

$$M(t) = 100heaviside(\sin\left[\frac{2\pi(t+6)}{24}\right]) \qquad \text{Equation 11}$$

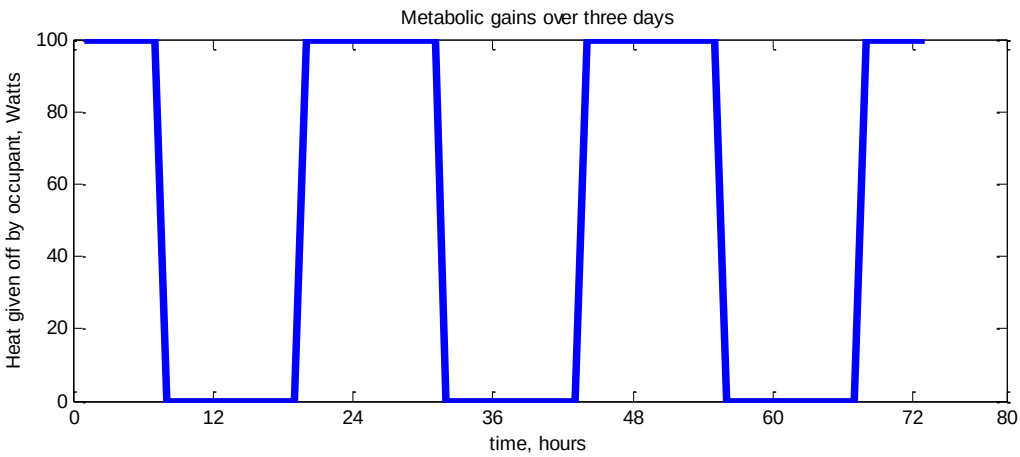

Figure 7: Metabolic gains function

Equipment gains will be added to this as 50W whenever the occupant is in.

Solar gain

It was decided that solar gain should be represented by the following mathematical description:

$$S(t) = 250heaviside\left(\sin\left(\frac{2\pi(t-6)}{24}\right)\right)\sin\left(\frac{2\pi(t-6)}{24}\right)\sin\left(\frac{2\pi(t+500)}{8760*2.23}\right) \qquad \text{Equation 12}$$

Figures 8 and 9 show the behaviour of the function S(t):

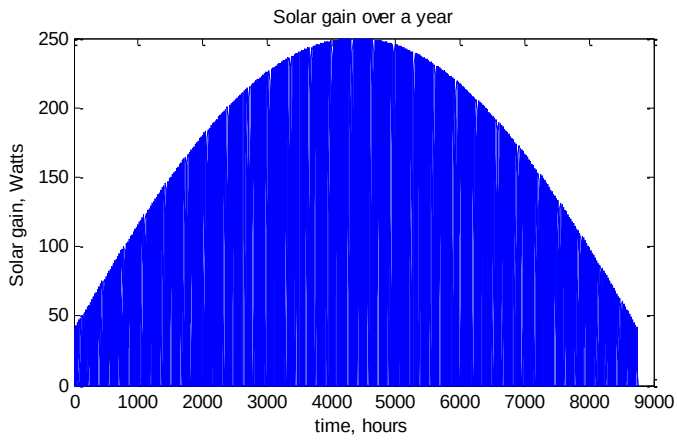

Figure 8: Solar gain function - annual profile.

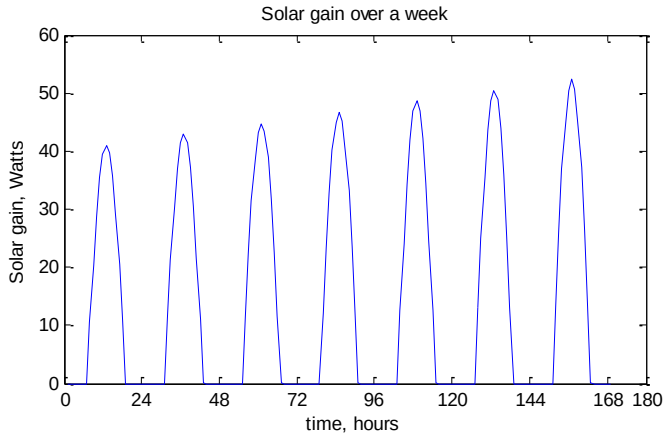

Figure 9: Solar gain function - weekly profile.

Note that limitations of S(t) are numerous, including that it is sunny every single day, from 6 a.m. to 6 p.m. Please see Table 8 which describes further limitations.

External temperature

$$To(t) = 5 + 16\sin\left(\frac{2\pi t}{2\ x\ 8760}\right) + 5\sin\left(\frac{2\pi(t+500)}{8760\ x\ 2.23}\right)\sin\left(\frac{2\pi(t+15)}{24}\right) \qquad \text{Equation 13}$$

This looks very complicated. This is because, when observing the TAS weather database, the author saw that there appears to be daily temperature amplitude, yearly variation in daily amplitude, and yearly amplitude. It was pointed out by Lowe, 2010 that there is also variation on a scale of about 4 days – this is partially addressed in Table 8.

Graphical representations of $T_o(t)$ are shown in Figures 10 and 11.

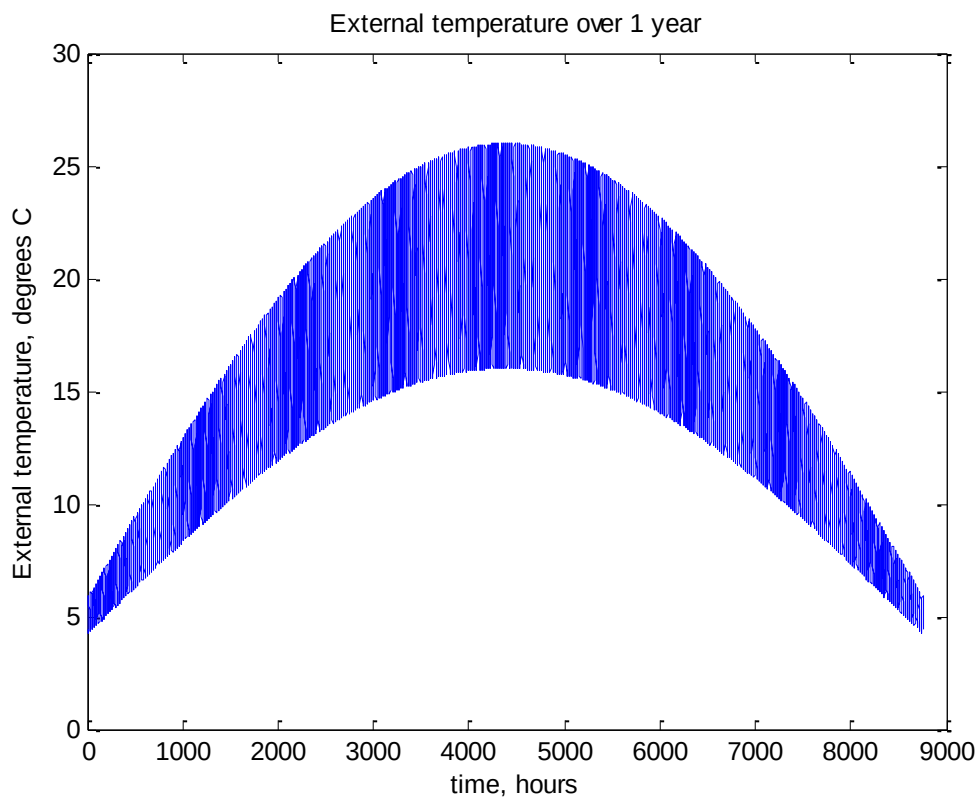

Figure 10: External temperature function - annual profile.

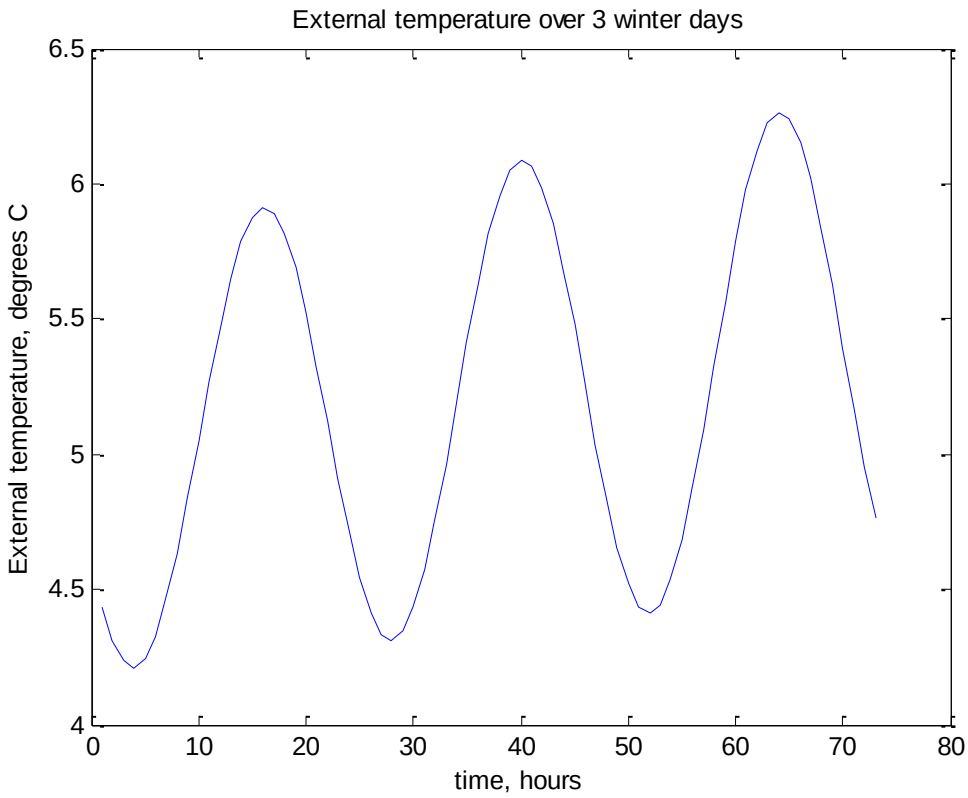

Figure 11: External temperature function - 3-day profile.

4.4. The final equation

Putting Equations 11, 12 and 13 in Equation 10, we have the finalised free-floating equation:

$$\frac{dT(t)}{dt} = \frac{M(t) + S(t)}{C} - \frac{HT(t)}{C} + \frac{H}{C}To(t) \qquad \text{Equation 14}$$

Where:

$$M(t) = 150heaviside(\sin\left[\frac{2\pi(t+6)}{24}\right]);$$

$$S(t) = 250heaviside\left(\sin\left(\frac{2\pi(t-6)}{24}\right)\right)\sin\left(\frac{2\pi(t-6)}{24}\right)\sin\left(\frac{2\pi(t+500)}{8760*2.23}\right);$$

$$To(t) = 5 + 16\sin\left(\frac{2\pi t}{2\ x\ 8760}\right) + 5\sin\left(\frac{2\pi(t+500)}{8760\ x\ 2.23}\right)\sin\left(\frac{2\pi(t+15)}{24}\right).$$

In Chapter 5, this equation will be validated and used with two objective functions.

5. FROM AN EQUATION TO AN EXPERIMENT

5.1. Validating the equation: the temperature profile

Equation 14 cannot be validated numerically as we are simulating a hypothetical dwelling. However, other methods of validation are *observation* and *detection of the presence of expected effects*. To validate Equation 14, the temperature profile across the year was observed for a lightweight and heavyweight building. The graphical results were compared to expectations, of which there are 2 main ones: that the heavyweight building would cause a smaller range of free-floating temperatures than the lightweight one, and that the heavyweight building's daily profile is displaced from the lightweight one by a time lag.

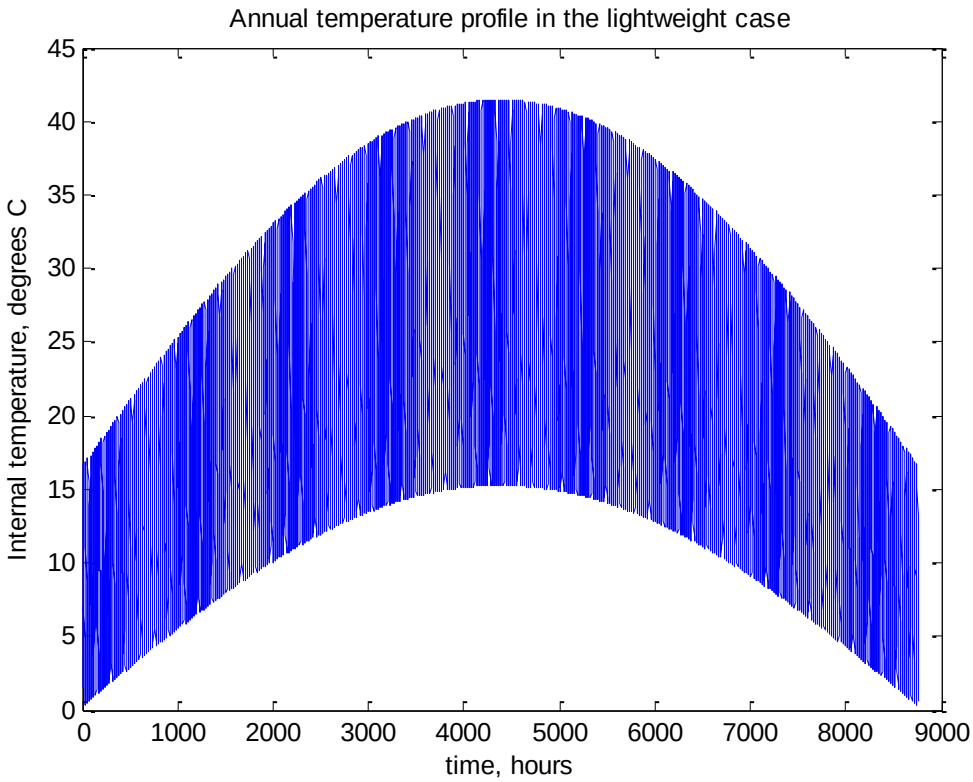

Figure 12: Annual temperature profile in the lightweight case.

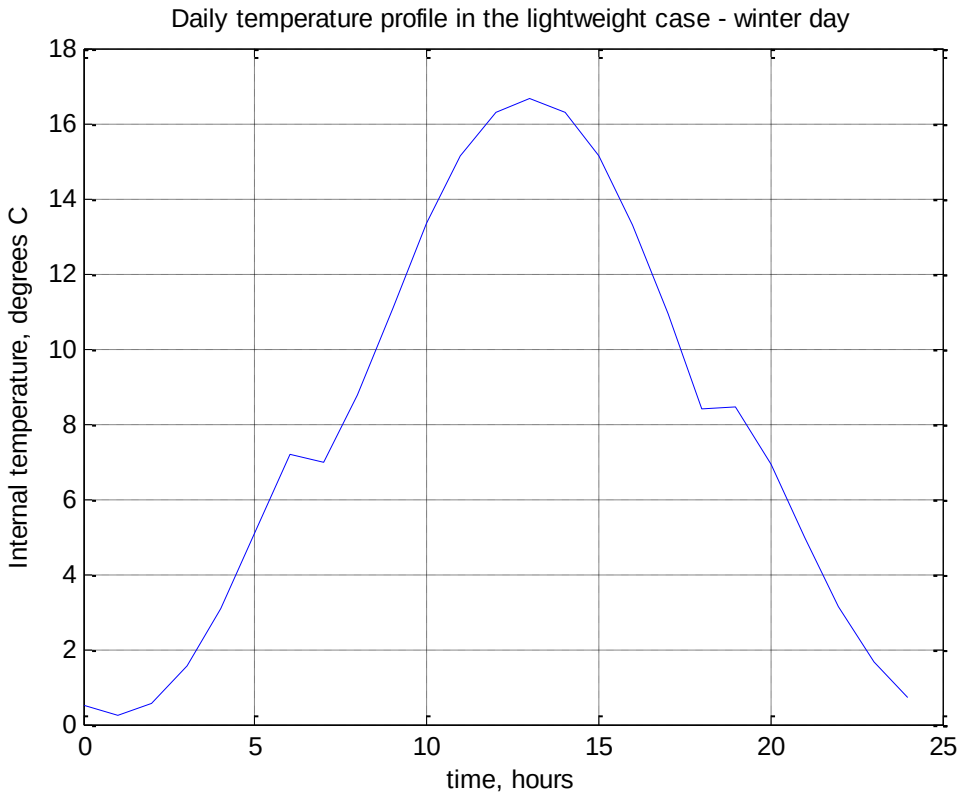

Figure 13: Daily temperature profile in the lightweight case - winter day.

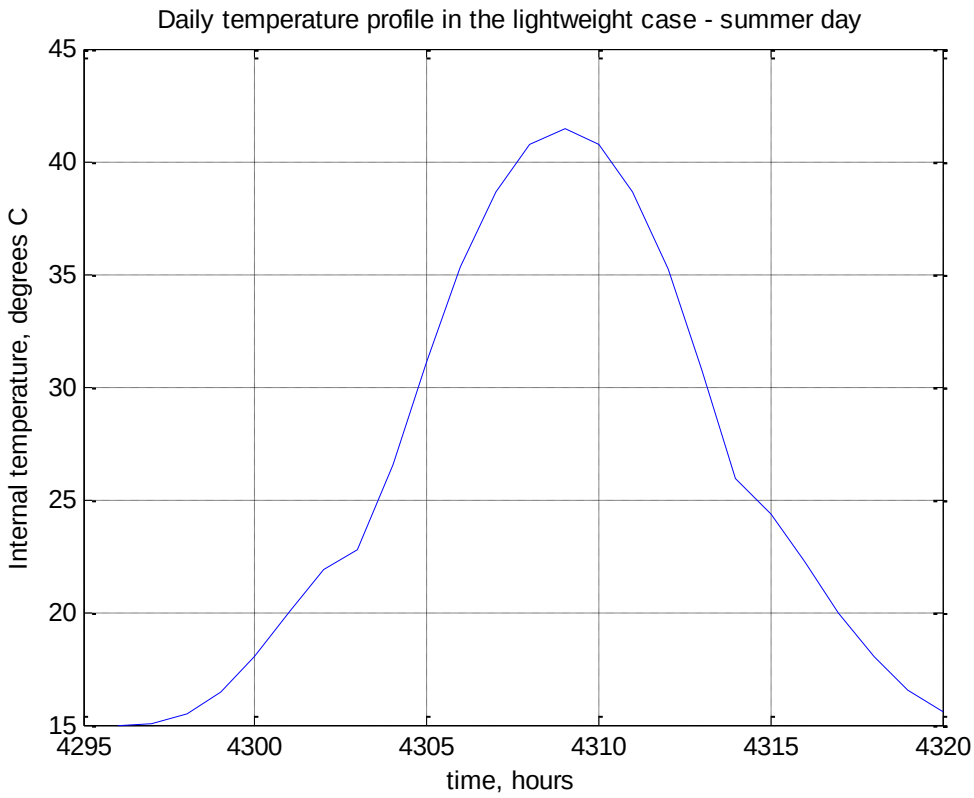

Figure 14: Daily temperature profile in the lightweight case - summer day.

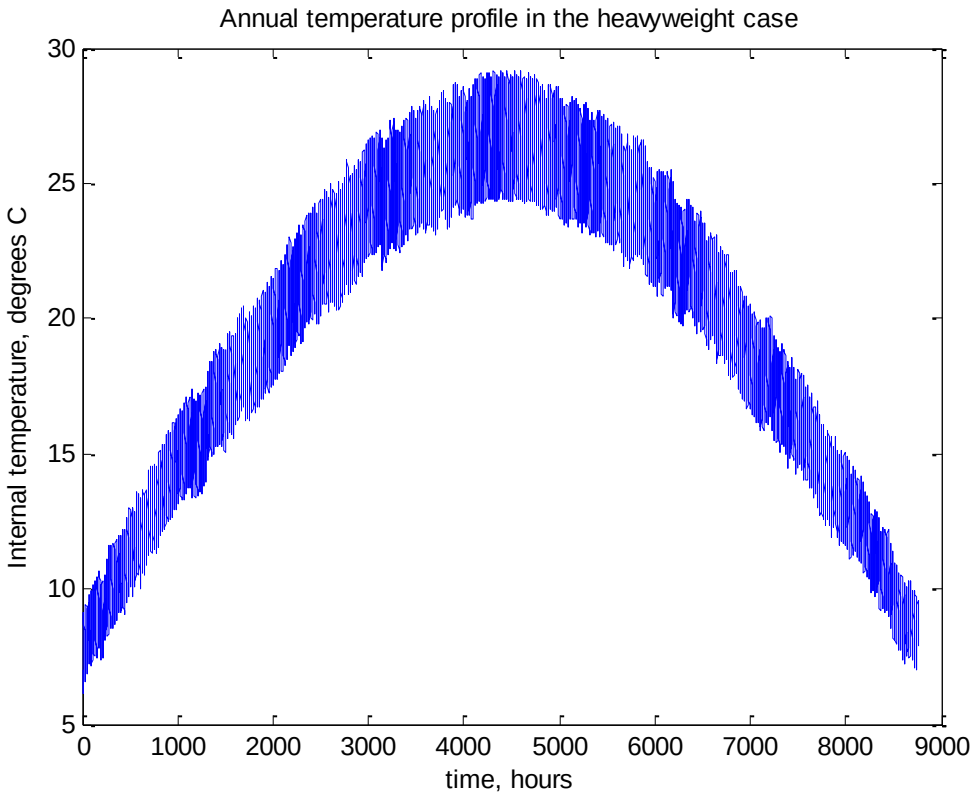

Figure 15: Annual temperature profile in the heavyweight case.

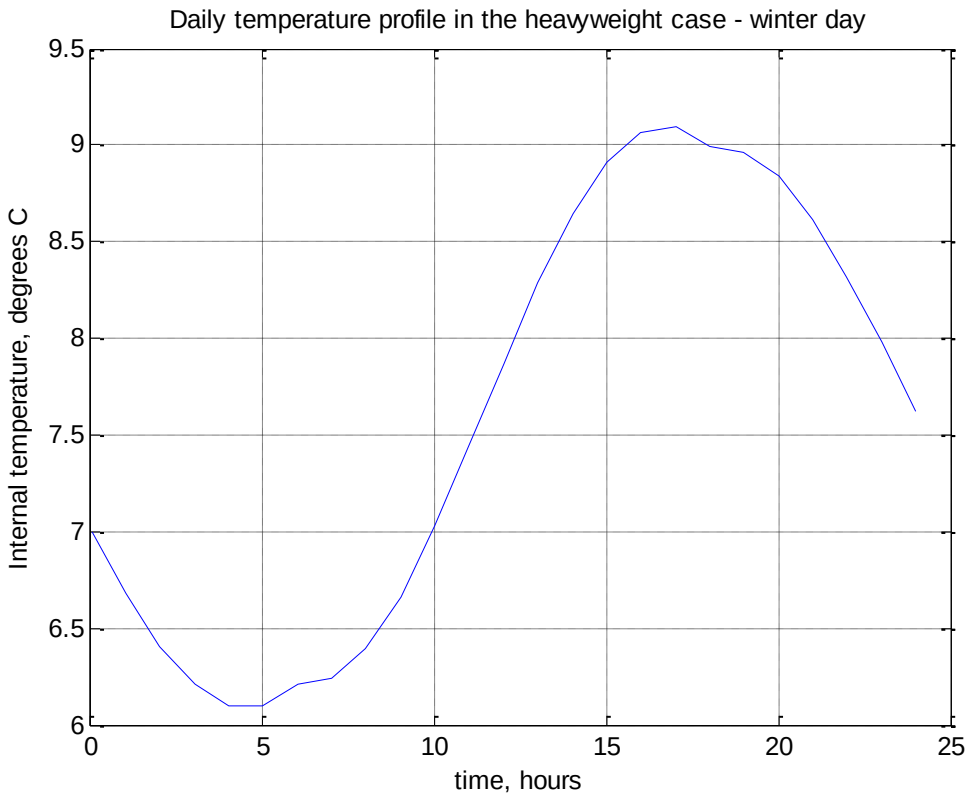

Figure 16: Daily temperature profile in the heavyweight case - winter day.

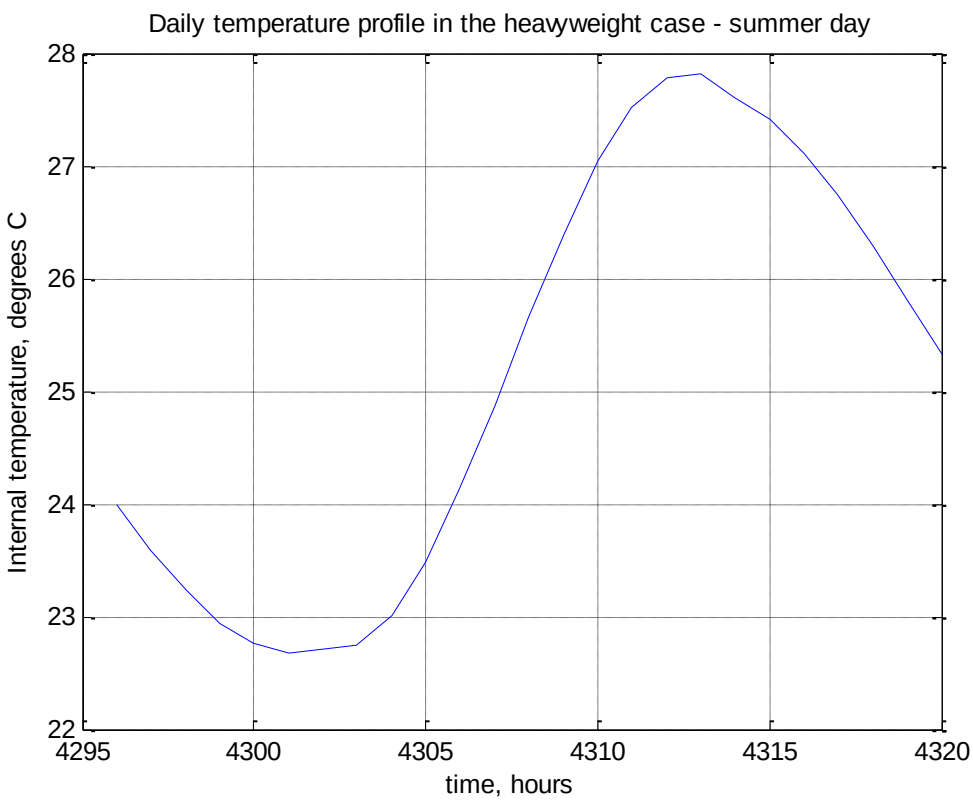

Figure 17: Daily temperature profile in the heavyweight case - summer day.

Figures 12 and 15 show the range decrease effect, and Figures 14, 15, 16 and 17 illustrate the expected time lag. Actually, the profiles were not so realistic the first time round, so modifications were made to the input functions that had not been spotted before.

5.2. Objective function of heating and cooling degree hours: the benefits of thermal mass

To contrast with the negative effects of thermal mass as shown in Chapter 3, the positive, temperature-smoothing effects can be demonstrated by a simple Matlab program which takes the free-floating temperature profile and works out the number of heating and cooling degree hours for each thermal mass (for occupied hours only). Not long will be spent narrating this section, since the above is only an intermediate objective function before the final one is defined later. However, it is worth including as it makes us discuss another assumption, which is definition of the 'comfort zone', whilst keeping the dwelling's temperature profile in free-floating mode for a while longer: this is a

more mentally checkable state than the one coming in the final model which adds yet another layer of complexity.

Definition of the comfort zone

Since the simulated dwelling just has one room, it is difficult to know what the accepted thermal comfort conditions should be – that of a bedroom, a living room, or otherwise? For simplicity, CIBSE Guide A shall be used to create just two criteria:

- The upper temperature limit shall be 28°C (CIBSE Guide A section 1.4.2.1, assuming the adaptive approach even though the dwelling has heating and cooling facilities);
- The lower temperature limit shall be 20°C (CIBSE Guide A Table 1.5, averaging over the types of rooms in a dwelling).

The program therefore evaluates the free-floating internal temperature for each hour of the year, and calculates any difference between this temperature and the boundary of the comfort zone defined above. The result, for the default input parameters stated in Table 6, is shown in Figure 18. Sensitivity analysis is left to Chapter 6.

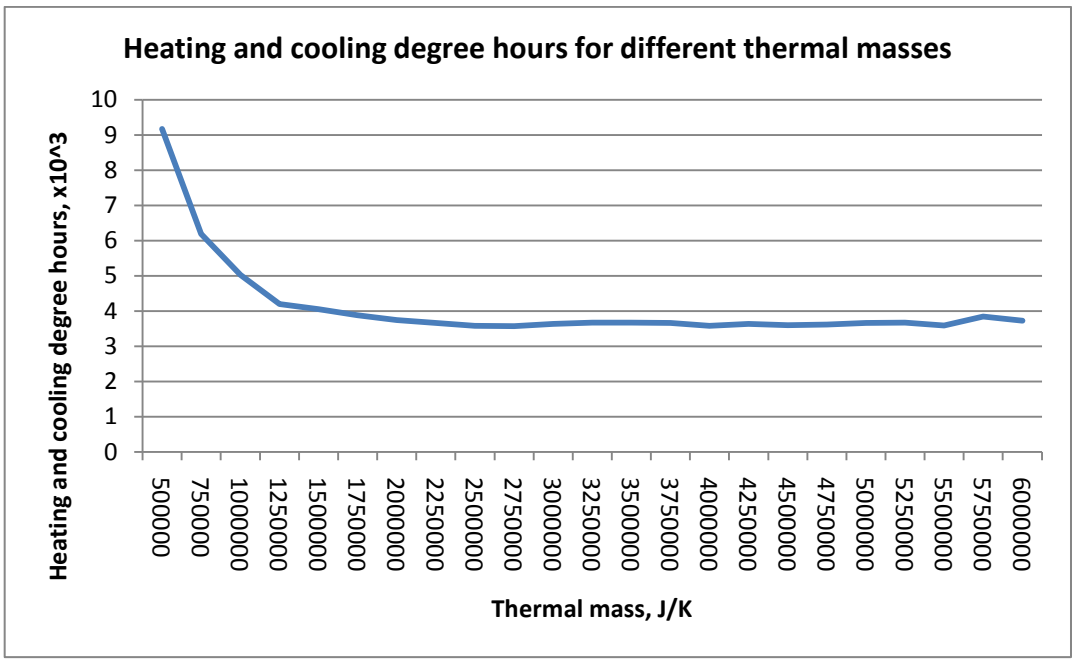

Figure 18: Illustration of benefits of thermal mass.

Table 6: Default values of input variables.

Input variable	Value unless otherwise stated
Dwelling dimensions	5 m x 5 m x 2.5 m
U	0.2 W/m^2K
n	0.5 ach^{-1}

As will be shown properly later, a lightweight building corresponds to the left hand side of Figure 18, and heavyweight is from thermal mass = 3,000,000 J/K to 6,000,000 J/K.

It can be seen from Figure 18 that heating and cooling degree hours can be reduced significantly by the use of thermal mass. The vague term ‘significantly’, as opposed to any numerical estimate, is used here - and the rest of the thesis is written in the same vein. This is because, as Chapter 6 will further discuss, such a model does not really produce reliable numerical answers, but instead trends and other visual phenomena.

However, this does not tell the whole story, and “heating and cooling degree hours” is not ultimately a good objective function to use, since the energy needed to heat (or cool) by/for one heating (or cooling) degree hour is not constant. Also, this objective function neglects preheating. But the model has been further validated by demonstrating that it shows the expected helpful effect of thermal mass.

5.3. Progress made, and next steps

So far in this chapter, the equation has been visually validated and incorporated into an initial program to evaluate an objective function across different thermal masses. The benefit of thermal mass, in terms of reducing heating and cooling degree hours, has been demonstrated. Next, this effect will be combined with intermittency, to create a more useful overall objective

function and gain insight into when thermal mass is a help and when it is a hindrance.

Unlike the temperature profile created in Chapter 4, the temperature of a modern dwelling cannot be free-floating all the time. The control system should maintain the temperature within the comfort zone during all required hours, yet let the temperature be free-floating during unoccupied hours to simulate the heating/cooling being off, then preheat again before the occupant(s) return(s).

5.4. Logic of the new program

Let us reconsider Figure 1:

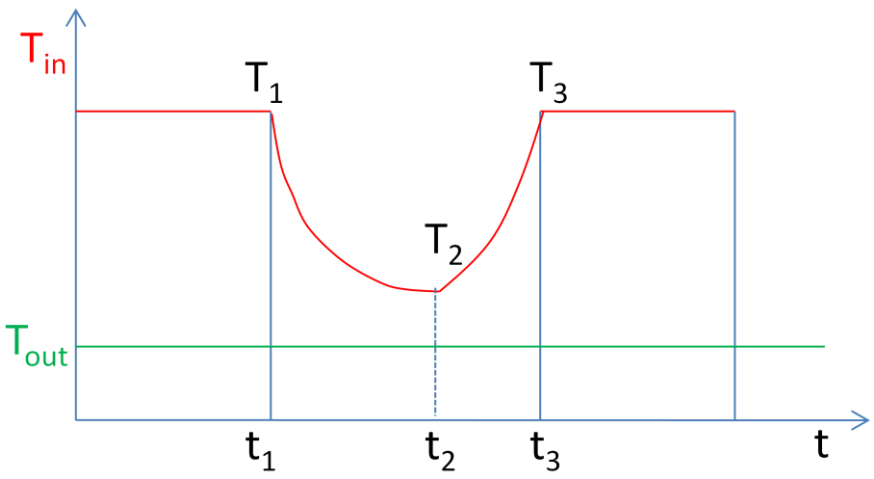

Figure 1 (repeated): Intermittency.

This is generalised in Figure 19 to mimic a heating and cooling system which reacts to any external conditions. Only two extreme examples are shown in Figure 19, because they are easiest to draw.

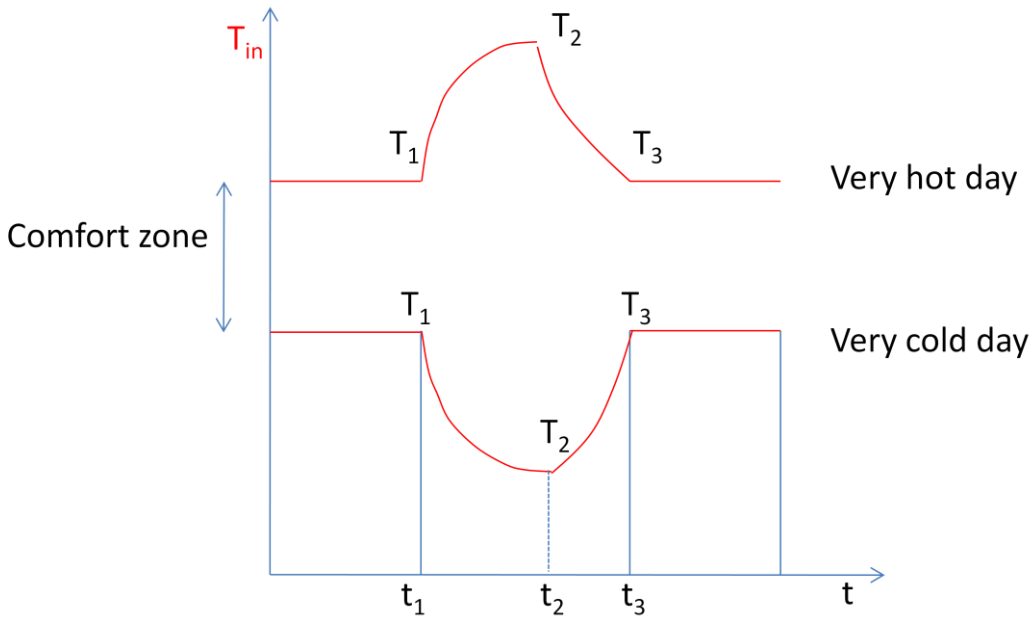

Figure 19: Two extreme examples of heating control system

In the drawn examples, the free-floating temperature is never naturally in the comfort zone. Between these come all the situations when free-floating temperature is in the comfort zone without the heating or cooling system being on.

Since the new integrals producing the curves in Figure 19 are not obtained analytically, the point t_2 (when the heating or cooling should come on) cannot be found using simultaneous equations this time. t_2 will therefore be set as one hour before the arrival of the occupants, and the heating/cooling will have variable power. (It should be noted that if t_2 were variable, this would entail the heating/cooling system knowing, at time t_1, all the solar gain and outside temperature for the following hours until t_3 – that is, the future weather! This has apparently been attempted but would not be accurate.)

Given a fixed t_2, T_2 can be found from solving the equation of the first curve, then T_2, T_3, t_2 and t_3 can be used as initial conditions to find the heating power (G) needed to get from T_2 to T_3 in one hour. This happens during what shall be named the 'pretempering' hour.

Each hour is of a certain type: occupied, unoccupied or pretempering. The algorithm must test each hour to see what type it is, and perform a different calculation in each case. This is a summary of what happens:

Table 7: The logic of the final program.

Type of hour	Temperature restriction	Energy calculation performed by the program
Occupied hour (for the control experiment, occupancy is between 6 p.m. and 6 a.m. every day)	T must be kept between 20°C and 28°C, whichever is the most energy-efficient.	The program calculates the energy needed to maintain the temperature within the comfort zone, by calculating how much the internal temperature would fall/rise by given an initial condition of 20/28, and the free-floating equation.
Unoccupied hour (for the control experiment, between 7 a.m. and 5 p.m. every day)	The temperature is allowed to free-float.	No energy is used.
Pretempering hour (for the control experiment, from 5 p.m. to 6 p.m. every day)	The temperature starts from where it was at the end of the free-floating period, and if that is below/above the comfort zone, T must end up at the lower/upper end of the comfort zone respectively, at the end of the hour. If T is already within the comfort zone, it is left to free-float during the preheating hour.	The program calculates the power, maintained over an hour (therefore, the energy) needed to bring the internal temperature to the desired level.

5.5. Writing and running the program

A program carrying out the above algorithm was written and run in Matlab. Its objective function is overall annual energy use over a continuous range of thermal masses, and optionally over a chosen second parameter.

To obtain this overall energy use, the program sums energy needed for each hour, over all the hours in a year. The hourly time-step was chosen because it is large enough for the wall-air interaction to be negligible (see Appendix 2) yet small enough for mass effects to be well-defined. However, well-defined effects come at a cost: the yearly profile for one thermal mass takes about 80 seconds to obtain. Over a suitable range of thermal masses, this becomes half an hour, and for, say, 5 values of a second parameter the simulation takes a number of hours. This long running time fixes the identity of the Matlab program: it can never really be attached to a user interface as something to 'play with' in real time. Please see Appendix 1 for a possible way to get around this problem via parametric analysis.

However, 80 seconds for one objective function evaluation is not actually bad given that some building simulation programs take hours to run once. The shorter the running time, the more experimentation can be carried out.

The Matlab code is displayed in Appendix 3. Included is an explanation of the algorithm to calculate the pretempering power.

Chapter 6 contains the simulation process and results.

6. RESULTS

The program described above was run for realistic values of input parameters as a “control experiment”. After the features of the results are understood, changes can be made to the input parameters and more simulations run to identify trends. As was alluded to in chapter 5, numbers are not given as results related to the real world, as their reliability is questionable. Tables of numbers are not presented either. Instead, the reader is encouraged to look for trends on graphs.

6.1. Control experiment

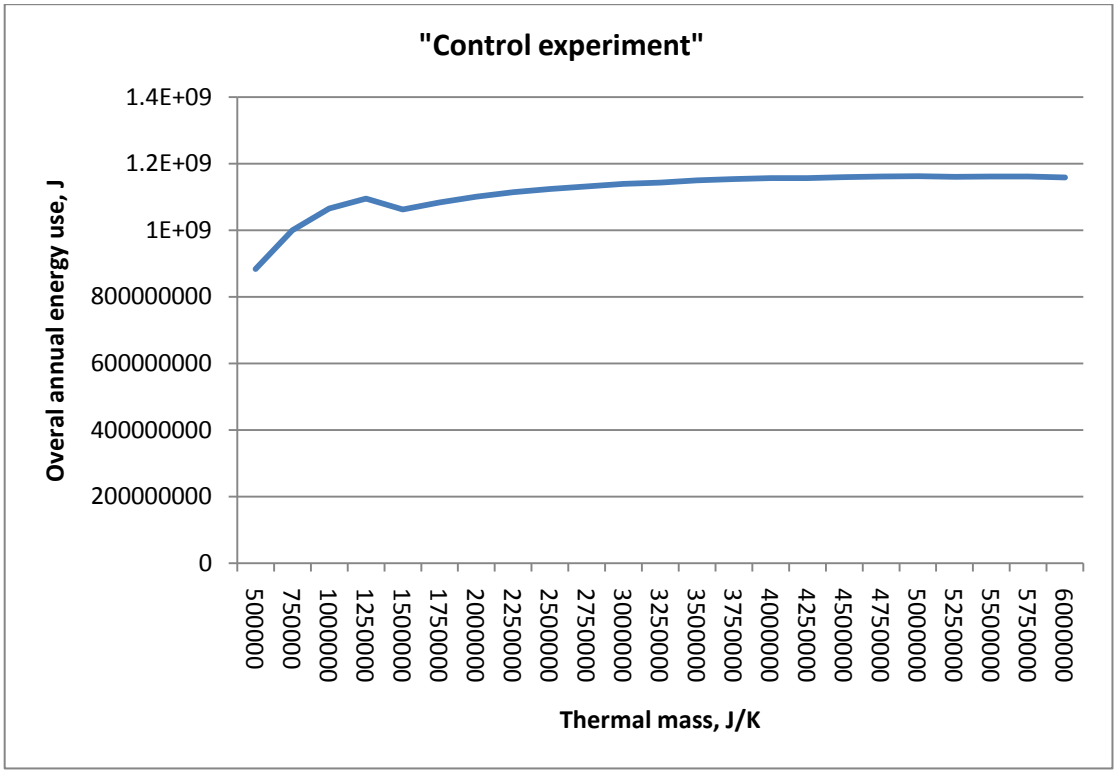

Figure 20 : Control experiment

Before saying anything about the shape of this graph, it is useful to highlight four general features of the graphs to be presented:

Firstly, to label which are the regions with and without thermal mass, and how far along the x-axis the effect of ‘clutter’ (see chapter 3) extends.

It was decided that the thickness of intentional thermal mass should be between 100 mm – the smallest realistic thickness of slab that can be pre-cast; and 200 mm – mass thicker than which could not be treated as isothermal. These are the 'constraints' on the system that were introduced as a concept associated with optimization in chapter 2.

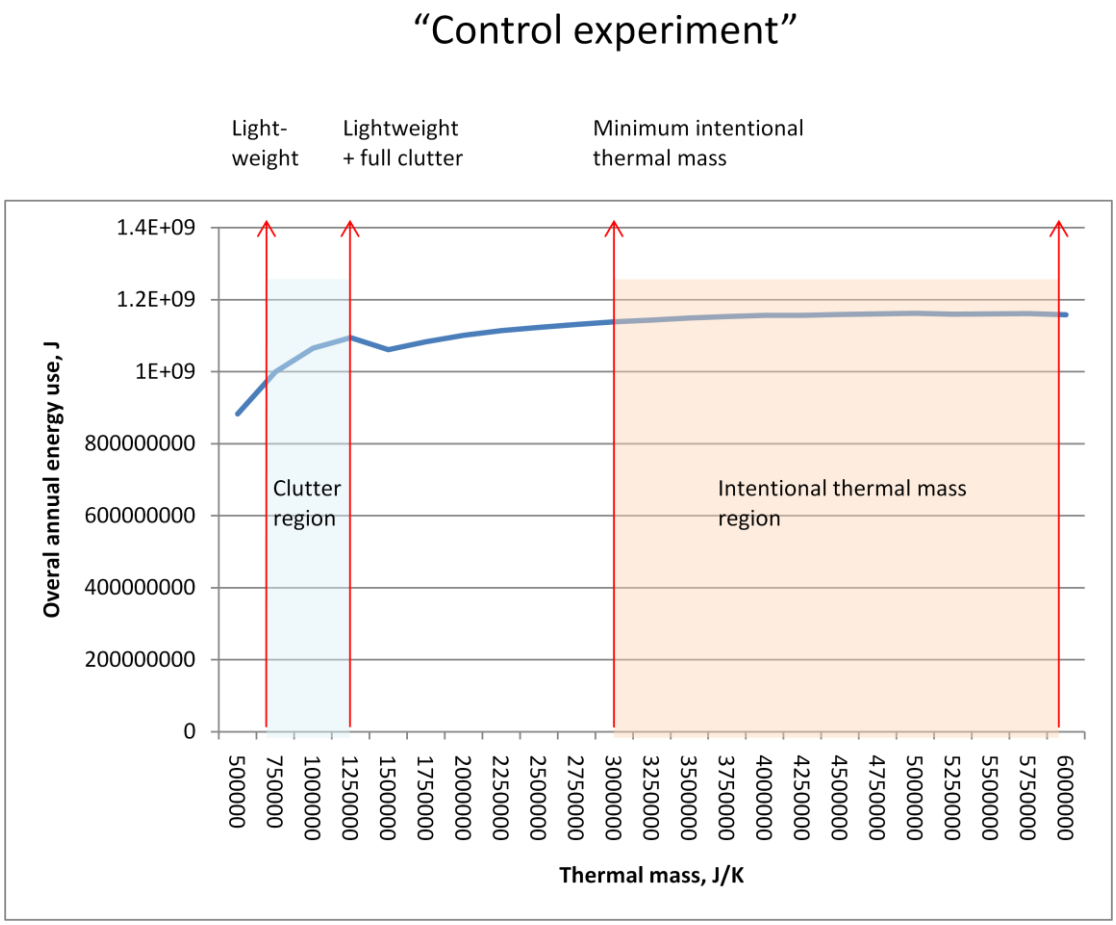

Figure 21: Showing regions of interest on the graph.

This will be the format of graph used for the remainder of the results section, with blue sections of the graph indicating the clutter region, and pink indicating the intentional thermal mass region.

Secondly, before commenting on the shape of the curve, it is useful to split it into its constituent parts. The objective function is the superposition of annual heating and cooling energy use, whose individual curves are shown in Figures 22 and 23.

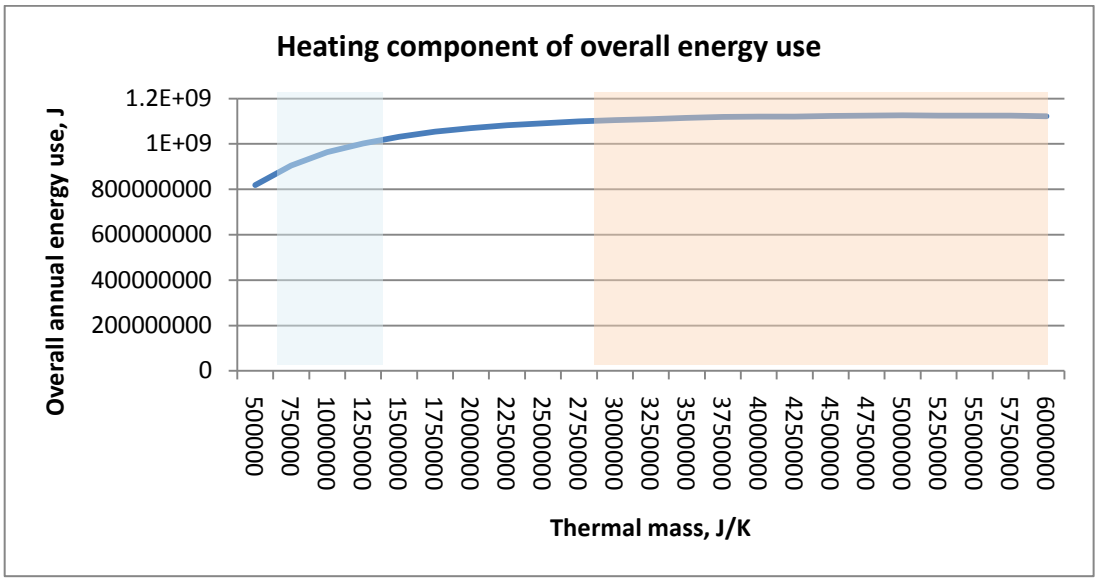

Figure 22: Heating component of overall energy use.

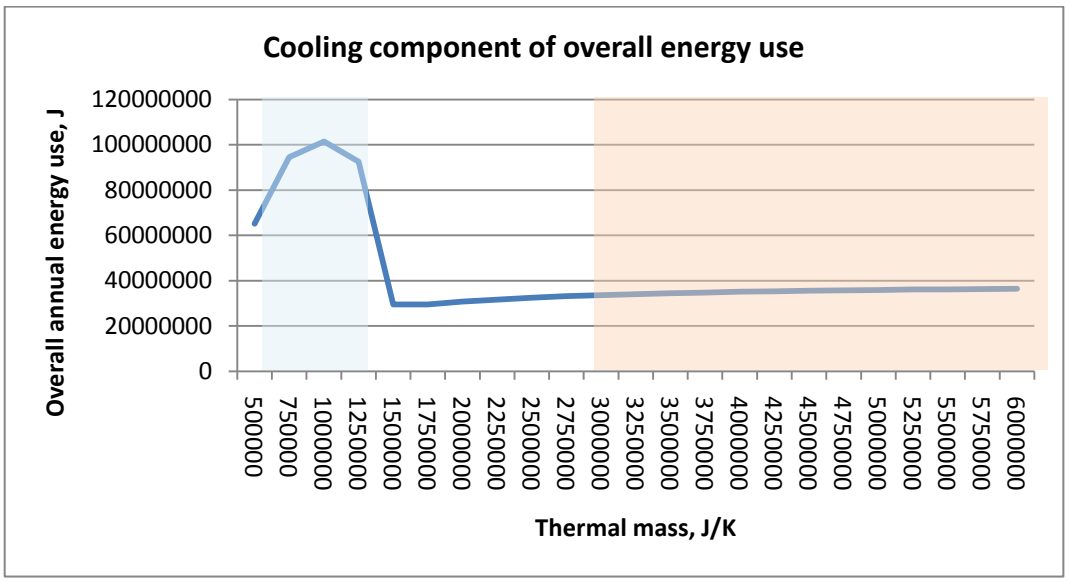

Figure 23: Cooling component of overall energy use.

Thirdly, it can be shown that the objective function is not smooth, by running a ‘zoomed in’ simulation of energy use over any small range of thermal masses. In fact, the domain of thermal masses was not arbitrary, it was taken from the intentional thermal mass (pink) region of the control graph, as then the effect of incrementing the thermal mass by a millimetre can be observed.

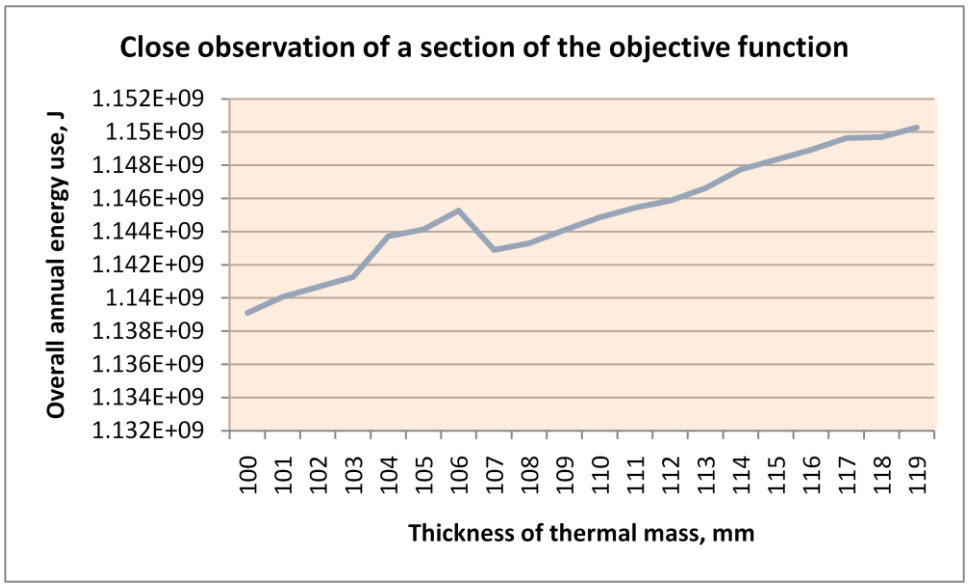

Figure 24: Close observation of a section of the objective function.

Fourthly, we need to know how energy use differs between lightweight and heavyweight cases, to see what is actually *causing* the shape of the objective function:

In these simulations, there are two kinds of heating/cooling. The first is that needed to maintain the temperature between 20°C and 28°C whilst the occupant is in (we will call this 'maintaining energy'); the second is that needed to preheat/precool the dwelling in the hour before the occupant returns ('pretempering energy').

For any two thermal masses, if the heating/cooling is on in both cases, there is no difference in maintaining energy. This is because the system is in a steady state, and mass has effectively no effect – since heat enters and leaves the mass at the same rate. This may seem obvious from physics, but the author had forgotten that it would happen until the results showed it, and wonders if it is commonly recognised by non-physicists/engineers.

Therefore, the only factors which can cause difference in overall energy use for two different thermal mass are:

- pretempering energy,

- maintaining energy – only in the case that a change in internal/external gains means that heating/cooling becomes necessary for the lower thermal mass, whilst the higher one manages to keep the internal temperature within the comfort zone.

Incidentally, this is a re-statement of the whole problem: balancing thermal mass' disadvantageous property of needing more pretempering energy with the advantage of keeping the internal temperature more pleasant during occupied hours.

How the overall energy use graph gets its shape

With the above four points in mind, let us examine the shape of Figure 21. As thermal mass increases from C = 500,000 J/K (lightweight and no clutter), the first thing that we observe is a relatively steep increase in overall energy use with thermal mass. This is due to the cumulative effect of increase in preheating and, predominantly, the increase in precooling (we saw this effect for preheating in Chapter 2). Then, the 'kink' in overall energy is due to a sudden drop in precooling from its maximum: at a given small range of thermal masses, the need for precooling suddenly decreases quite quickly.

It is interesting that the 'kink' occurs in the *clutter region* in this particular graph (and, it will be shown, in most other cases of input parameters). This shows that clutter can cause predictions of overall energy use to be out by a relatively large amount, and that spaces built to be deliberately lightweight may not perform in energy terms quite as expected.

After the kink, overall energy use slowly increases with negative acceleration. Precooling is zero, and the other types of heating and cooling all increase slowly.

Now that the curve is understood, the next thing to do is to vary input parameters in the equation and see if this curve maintains the same form or not. Before doing so, it is useful to discuss how the results should be presented.

How to visualise the model space

The differential equation underpinning the model (Equation 14) is shown again below.

$$\frac{dT(t)}{dt} = \frac{M(t) + S(t)}{C} - \frac{HT(t)}{C} + \frac{H}{C}To(t)$$ Equation 14 (repeated)

The following parameters can be varied:

- M (occupant gains – metabolic or other): variable magnitude
- H (heat loss coefficient) = UA + (1/3)nV: variable magnitude in two ways:
  - factors specific to heat loss – U and n – at constant A and V;
  - the building volume, which affects A and V, and thus affects C – the amount of thermal mass.
- (not in the equation, but in the program) variable occucpancy schedule.

Therefore, we have one dependent variable (overall annual energy use) and at least 5 independent variables. We shall call the set of all solutions the 'model space'. The model space is therefore 6-dimensional, and is illustrated in Figure 25.

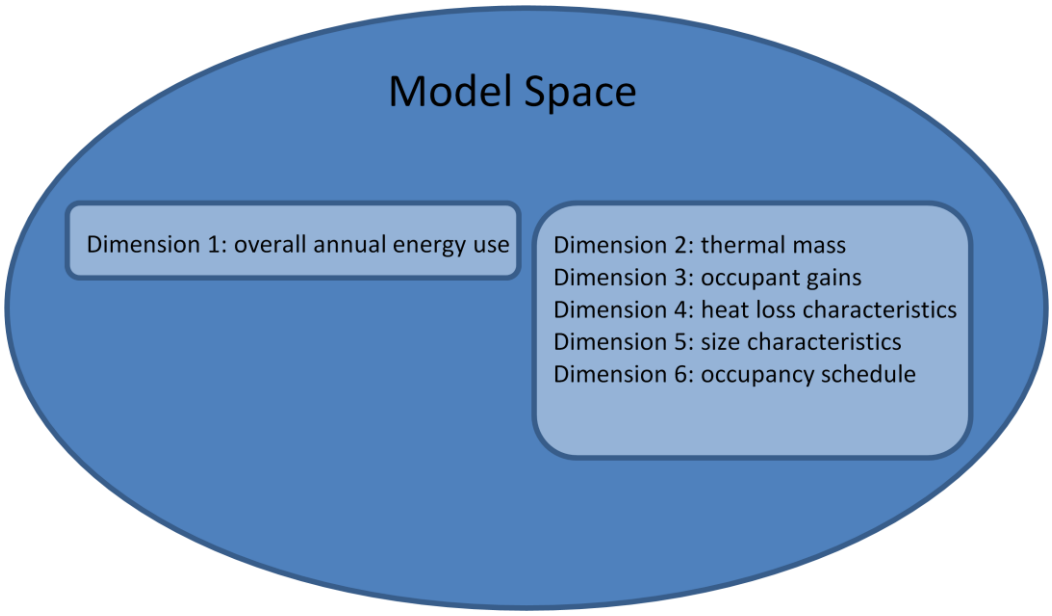

Figure 25: Illustration of the Model Space

Now, a six-dimensional model space is not easy to visualise. There is a three-fold problem of presentation of results:

- Displaying the results in a table in the clearest possible manner (see Appendix 1 for how this was done in Microsoft Access in a manner readable by Java).
- Displaying the results to the reader of this thesis and in general to mathematically-minded people.
- Displaying the results to a generally- (but not necessarily mathematically-) interested person.

The second of these challenges is attempted in this thesis by two-dimensional graphs showing up to three variables, sometimes displayed as a matrix. However, the author thinks this is less than ideal, and therefore in Appendix 1 there is an attempt at an alternative method of visualisation: a user-interface where the user can traverse the model space in certain directions.

Presentation of graphs

This is one point at which the sequence of this report will depart from the order in which tests were actually carried out. The first test presented, occupancy, was originally done last, and was found to be most important - so much so that the rest of the results were re-simulated and are presented in context of these findings.

The reader should note that the forthcoming results are not presented as a formal ‘sensitivity analysis’. This is because very little mathematics is used to compare the importance of varying different parameters; instead, visual inspection is mainly used. This, as mentioned before, is because the author does not trust her models, and therefore her philosophy is not to give absolute numbers but insight through trends. If a proper sensitivity analysis were to be undertaken, the method to be used could reflect that used by Breesch and Janssens, 2005; that is, global Monte Carlo analysis.

6.2. Modifying occupancy hours (and adding weekends) – development of scenarios

In chapter 2, it was pointed out that for variables whose values are unknown to the point they cannot be ascribed probabilities, scenario analysis can be used. In the case of occupancy hours, scenario modelling seems to be most suitable, firstly since occupancy is not really a continuous variable (unlike, say, thermal mass thickness) and is always changing, secondly because, except in special cases, those who design houses usually have no idea of the occupancy schedule of the resident.

Three scenarios were developed:

1. The occupant is out from 6 a.m. to 6 p.m. every day and in the rest of the time – perhaps reflecting a young person working in the City.
2. The occupant is out from 8 a.m. to 4 p.m. every day from Monday to Friday – perhaps reflecting a member of a family.
3. The occupant is out from 10 a.m. to 2 p.m. from Monday to Friday – perhaps reflecting a pensioner.

Figures 26-29 below show the differences in energy use in these scenarios, plus zoomed-in versions of each scenario:

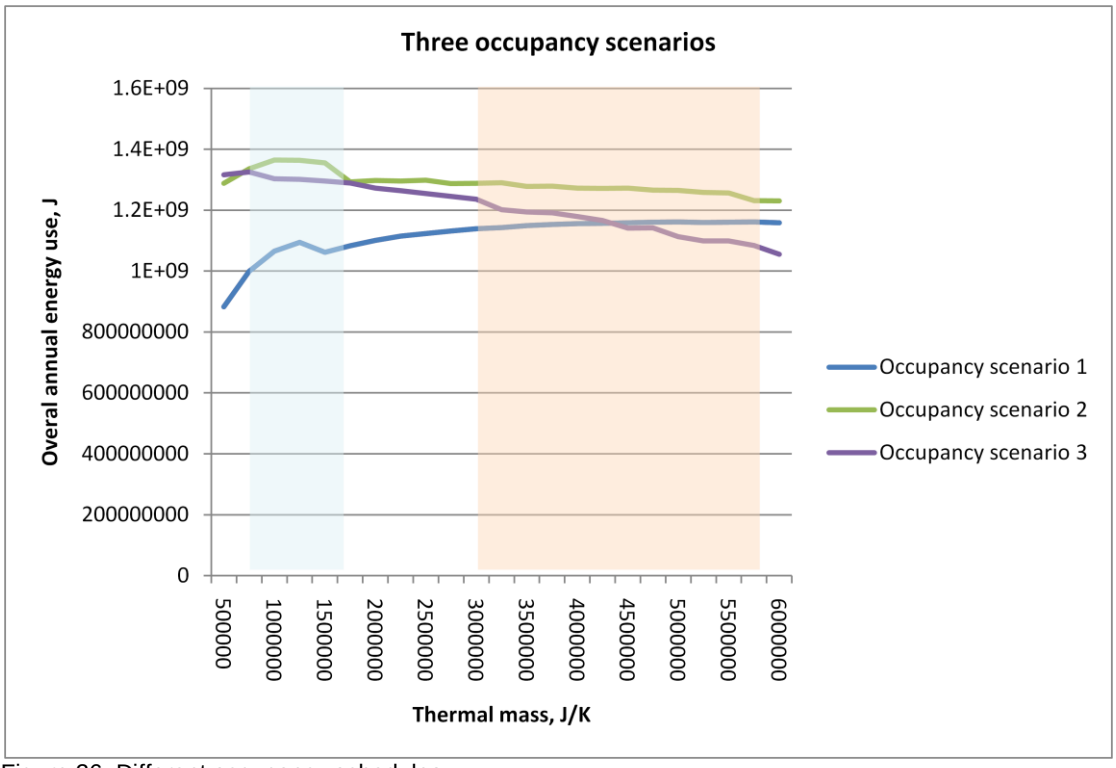

Figure 26: Different occupancy schedules.

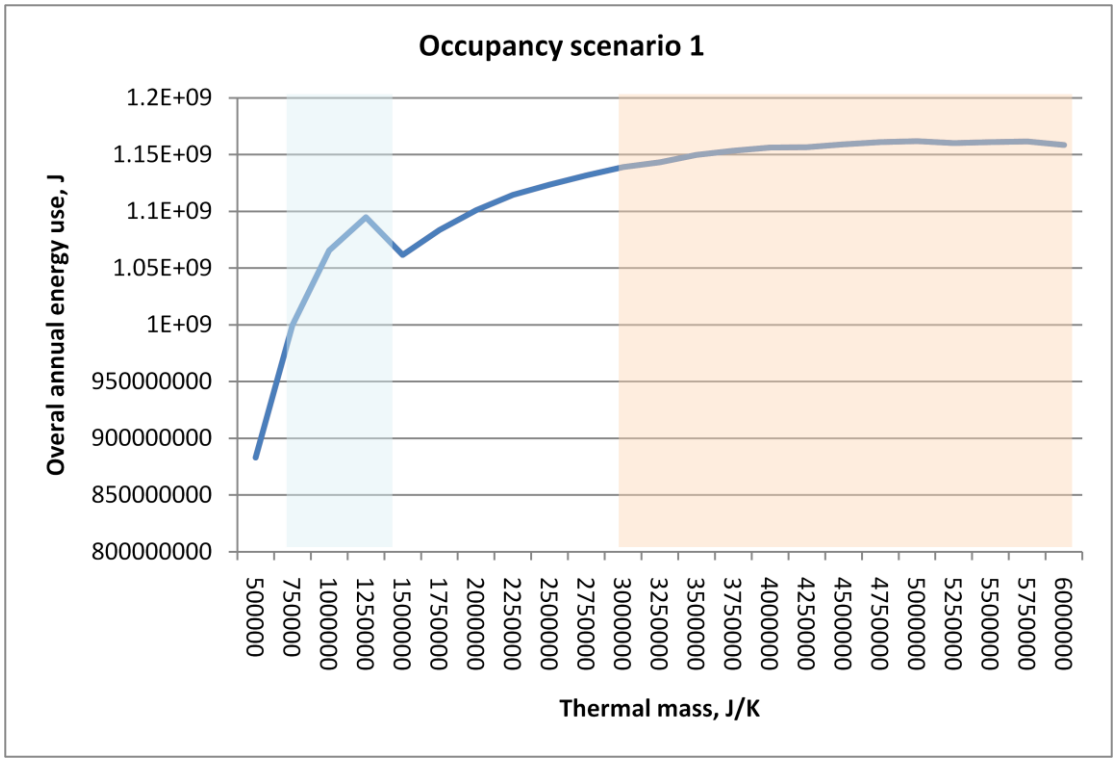

Figure 27: Occupancy scenario 1.

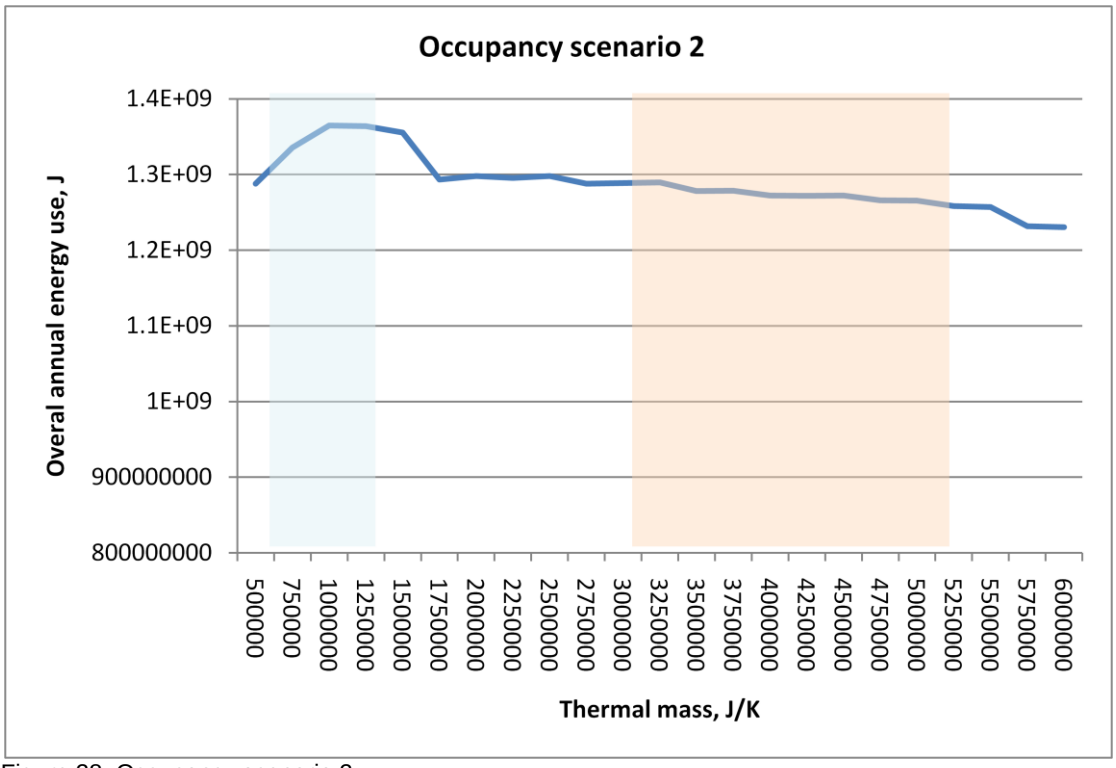

Figure 28: Occupancy scenario 2.

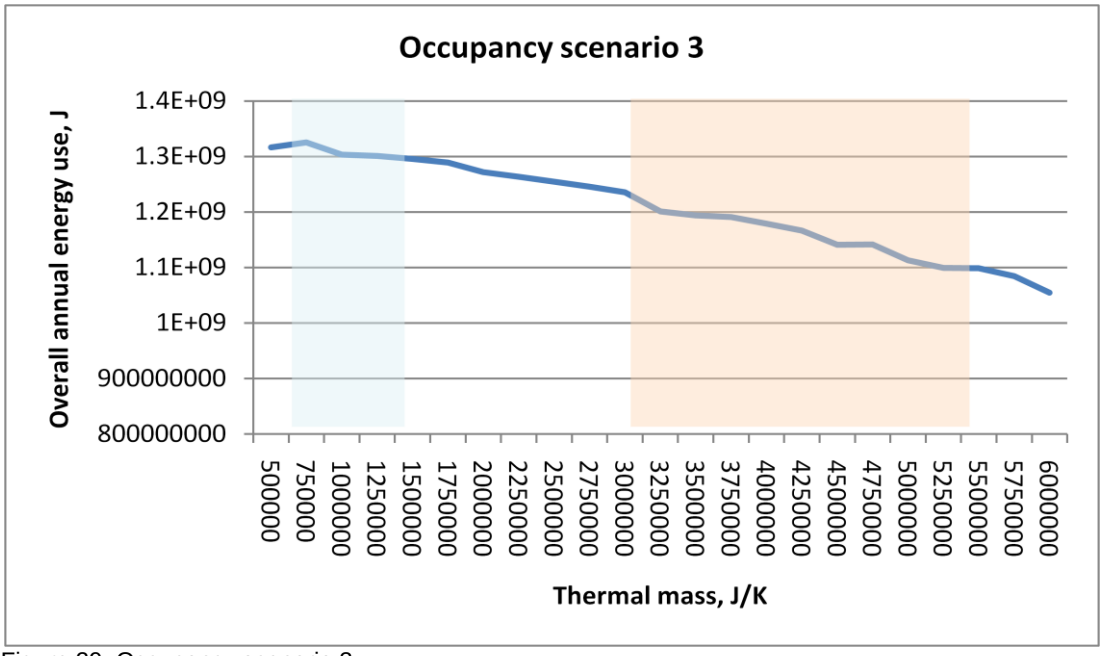

Figure 29: Occupancy scenario 3.

It seems that varying occupancy hours changes the sign of the gradient of the graph, thus causing a switch in optimum mass. For dwellings in which the occupant is out during the day, lightweight construction is better, but for those in which the occupant is there for most of the day, heavyweight is better (again, numbers or percentage changes will not be used).

This effect is what we would have predicted physically (more thermal mass being helpful when the occupants are in during the daytime, and being a hindrance when they are not), but we did not know that the switching point would manifest itself over this range of occupancy scenarios.

Further analysis of occupancy

It is interesting to ask: what is it specifically about occupancy that causes the shape of the graph to change so much between scenario 1 and scenario 3? There are two main differences between these scenarios. One is that in the latter, fewer hours are spent out (and therefore the building is in free-floating mode for less time). Secondly, the occupants are in during the hours of the day when overheating is most likely to occur. Which effect is greatest: the *relative* decrease in unoccupied hours, or the *absolute* times of day the occupants are in?

In fact, it is both. This can be shown without too much extra work. Firstly, it was shown in Figure 4 that increasing time spent out makes a heavyweight construction increasingly less favourable under almost all circumstances. Secondly, further simulations were run with the occupied hours delayed by 2 hours and then reversed completely, to see the effect; please see Figure 30.

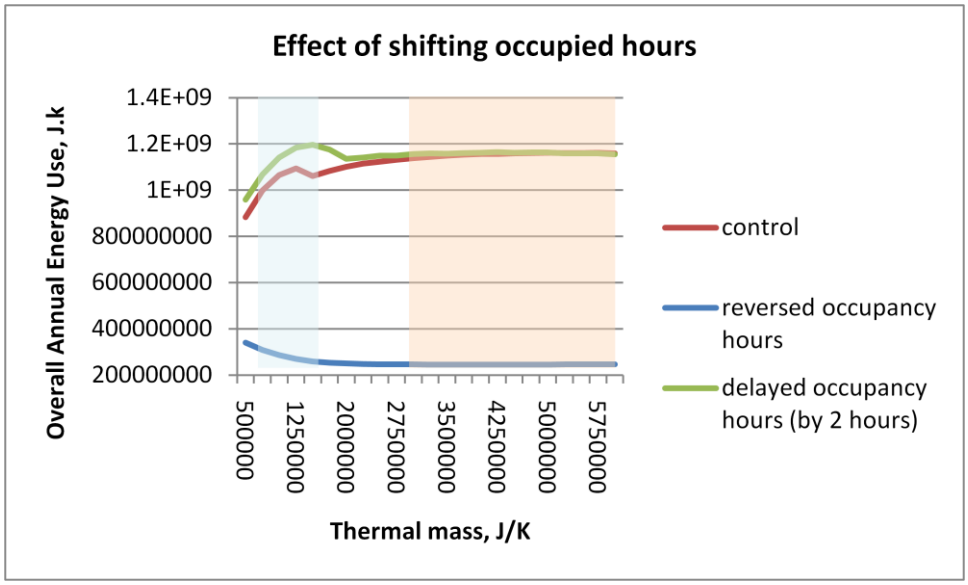

Figure 30: Effect on energy use of shifting occupied hours.

A shift of a couple of hours in the occupancy schedule does not have much effect, but reversing the schedule completely causes the curve to shift from heating-dominated to cooling-dominated.

Therefore, both the absolute and relative changes in occupancy schedule outlined above are important.

The rest of the informal sensitivity analysis will now be carried out *for each of scenarios 1 to 3*.

6.3. Varying internal gains

A term, “internal gains factor”, was invented to describe the extent to which internal gains exceed that given off as metabolic gains by one occupant - roughly 100W. Electrical equipment, other people, and phenomena such as boiler heat leakage are all potential contributions. The range tested was from 100W (internal gains factor = 1) to 300W (internal gains factor = 3). The results are shown on the next page.

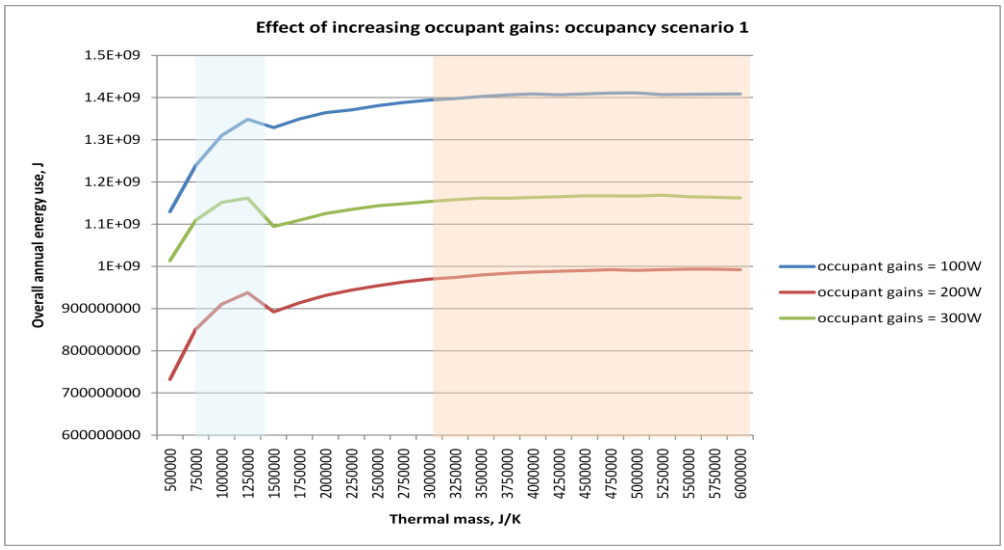

Figure 31: Increased occupant gains: 1.

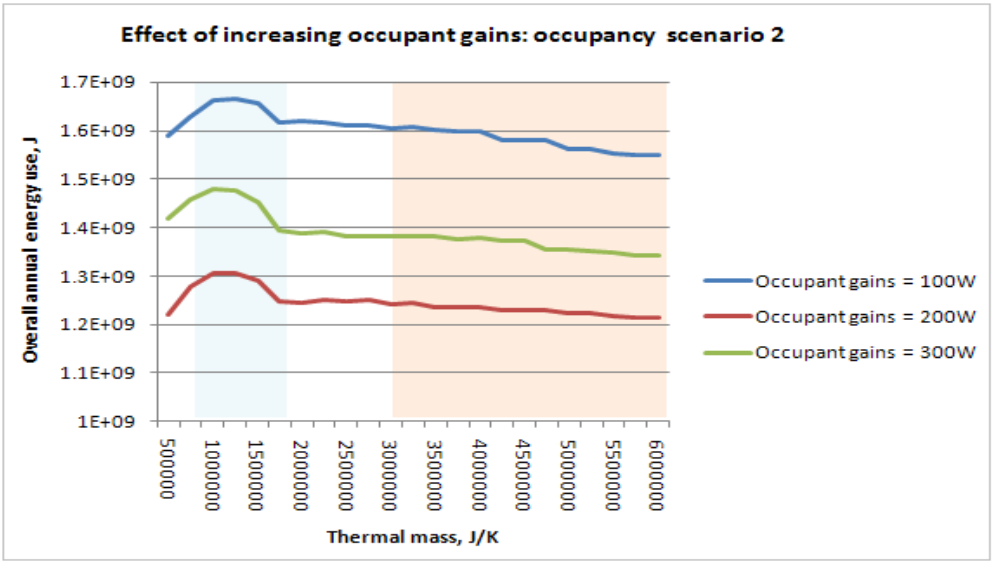

Figure 32: Increased occupant gains: 2.

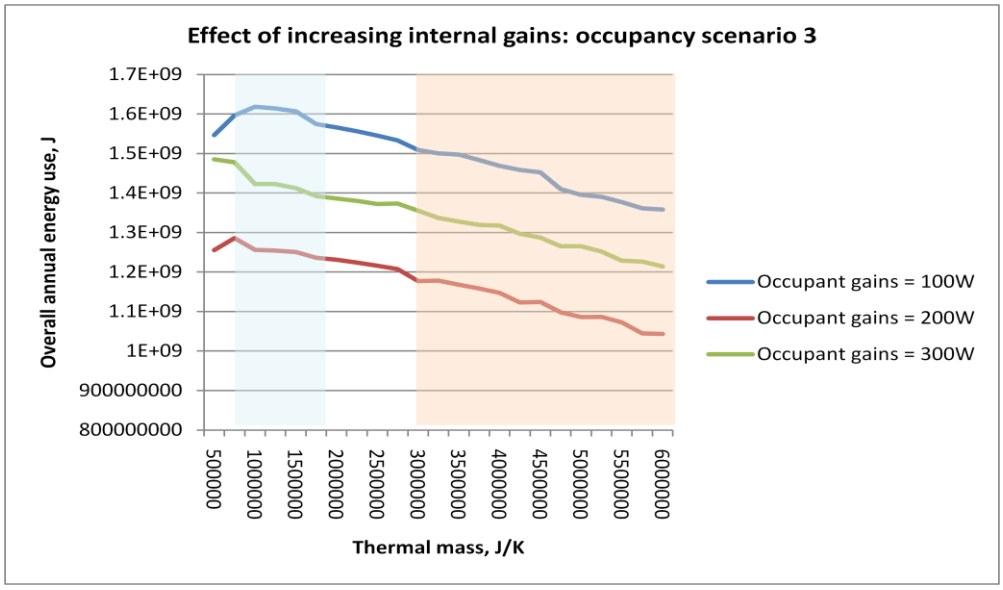

Figure 33: Increased occupant gains: 3.

An increase in occupant gains does not change the shape of the graph in general; it causes a linear translation in the y-direction. Therefore this effect can be described as ‘predictable’: even though it is not obvious whether a curve will be moved upwards or downwards with a change in occupant gains, the change in energy use over different thermal masses stays the same.

6.4. Varying building volume

In this experiment, the building volume was increased by a factor of two, four and eight.

It is not entirely helpful to plot overall energy use against thermal mass; more information can be gained from plotting energy use over volume scaling factor, against thickness of thermal mass. The first marker on the x-axis, 0.23..., is the equivalent thickness of concrete of a lightweight construction.

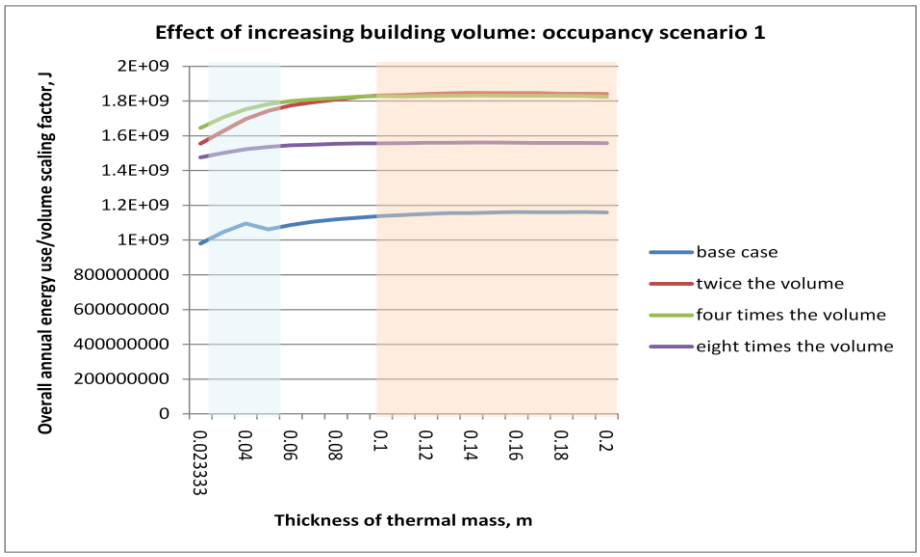

Figure 34: Increased building volume: 1.

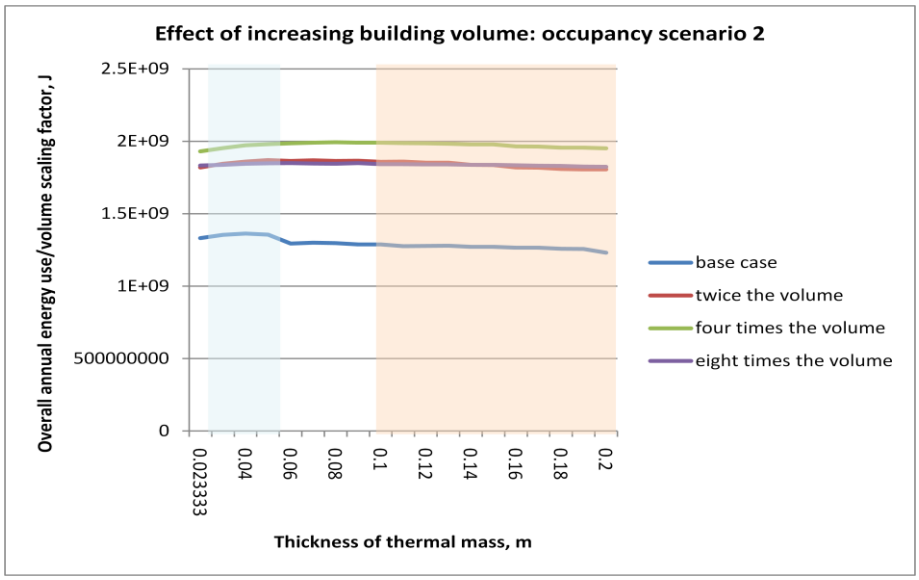

Figure 35: Increased building volume: 2.

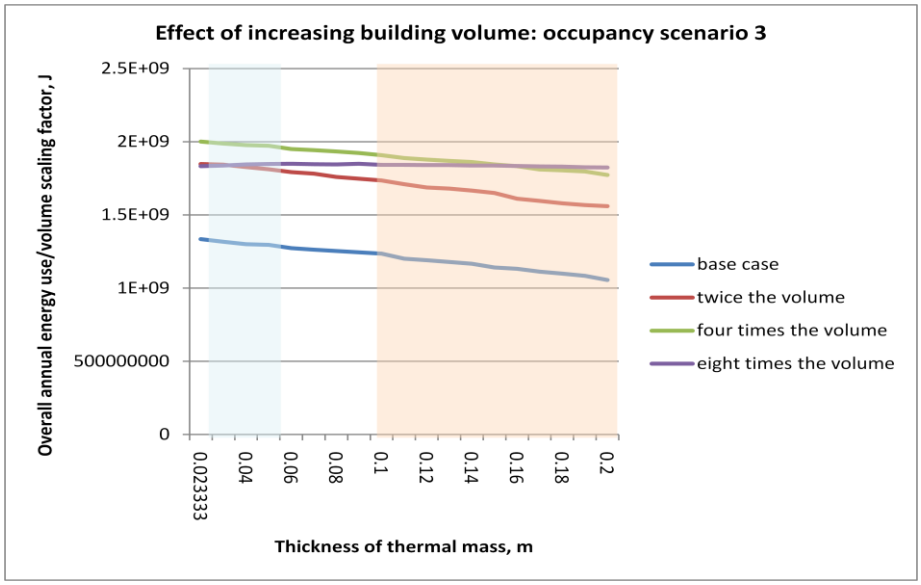

Figure 36: Increased building volume: 3.

6.5. Varying heat loss coefficient

So far, it does not seem that changing parameters causes anything to change differently between the three occupancy scenarios. However, changing the heat loss coefficient at constant building size causes some interesting effects, which will now be explored.

Specific types of construction can be tested here. The first one is super-insulated dwellings ($U \leq 0.15$ W/m^{2}K), for which it is sometimes claimed that high thermal mass is beneficial (see for example Airaksinen et al., 2006); this is tested at different air change rates ($n = 0.5$ to 1.5 ach^{-1}) to demonstrate airtight and non-airtight dwellings. Following this, non-super-insulated dwellings ($U \geq 0.2$ W/m^{2}K) are tested.

Since there are 3 independent variables in this particular test (occupancy, thermal mass, insulation/airtightness [categories, and individual values within those categories]), the results, Figure 37, are displayed as a matrix of graphs on a large sheet on the following page. This is so that trends can be seen across the graphs – vertically, horizontally; and also within each graph. In other words, to show intra- and inter- type trends. Note that the scales of the graphs are not all the same so some effects look bigger than others – the important thing here is the relative size of an effect.

PLEASE PUT THE A3 SHEET IN INSTEAD OF THIS PAGE ☺

Figure 37: Matrix of graphs

Observations from Figure 37 - the matrix of graphs

It is intuitive that super-insulated buildings have little point if the air change rate is too high (without any kind of heat recovery system). The graphs in Figure 37 demonstrate this in terms of thermal mass use: it is generally better to use thermal mass when insulation is thick and air change rate is low. However, the graph in the top left of Figure 37 shows that at low heat loss, the energy use is fairly unpredictable – see 'Predictability' below. When air change rate increases, thermal mass becomes a hindrance rather than a help, because all the energy put in (for example to warm the space) is then lost through convection. In other words, if the super-insulated building is not used 'correctly', its thermal mass could work against it.

The above is is an argument for using mechanical ventilation (to get good control) and heat recovery. The latter is not accounted for in this model, so the true Passivhaus cannot be put in to test.

Predictability

All of the above has been known and talked about by people for a while. However, the author has not seen any literature on the low-heat-loss end of the results: if pre-cooling is included in the building management system, as could happen in the future, the low heat loss dwellings modelled here show that the effect of clutter is very important and will render the energy use fairly unpredictable. For example, see the top left hand graph. The model seems to show that a few extra desks here could send the energy use either up or down by not far off an order of magnitude! Maybe this is not completely correct, and maybe that is why there is no literature on it, but is an interesting outcome of the model used in this thesis. It reminds us of Lowe et al. (1997) in which it is advised that it is safer to put in more rather than less insulation at thicknesses near the optimum, since the CO_2 penalty of being wrong is less.

As a result, perhaps it is better to build heavyweight dwellings, as the sensitivity of energy use with variation in thermal mass/clutter is not so great,

and thus the energy performance is potentially easier to predict. This is interesting: the author thought that the results presented in this thesis would be ‘thermal mass causes an increase/decrease in energy use’ but not ‘thermal mass causes energy use to be more predictable’. Is this a reason to recommend its use? Perhaps.

The author has not tested what happens if pre-cooling/maintaining cooling is not included in the model, and thermal comfort is traded off against energy use, because the objective function is less well-defined – and is a case of multicriterion optimization. However, this is probably quite important, so please see Appendix 3 to read about how it could be treated.

6.6. Assumptions and their predicted effect on the results

It is necessary at some point in the report to state explicitly the limitations of the model in terms of the assumptions made. This point seems suitable – after the presentation of the results but before the section in which we try to gain insight from them.

Table 8: Assumptions and their predicted effect on the results.

ASSUMPTION	PREDICTED EFFECT ON RESULTS
No zoning: in reality space heating/cooling demand varies with extent of zoned control system (see for Example Table 6-4 of Barrett, 1992).	If zoning is included, lightweight construction becomes more favourable since total occupancy hours are split into hours spent in each room, therefore more pretempering energy is used. Barrett (1992) calculates the reduction in energy demand between scenarios of low thermal mass and low thermal mass + zoning to be about 40%.
Boiler effects are ignored.	In reality, thermal mass can cause boilers to be more efficient, and hence lower overall energy use. This is because the thermal capacity of the house can save the boiler from switching on and off so much.

ASSUMPTION	PREDICTED EFFECT ON RESULTS
Weather varies daily and annually, but not on the scale of decay of a typical cyclone (the mesoscale, in meteorology): that is, a few days (Markowski and Richardson, 2010).	The same weather pattern is prevalent over a time of the same order of magnitude as the building time constant, e.g. no solar gain for a period in winter equal to the natural cooling period of the building. Different effects would occur for every weather pattern, and the results depend on whether the building genuinely does cool down or whether the occupants are in and therefore the heating is on.
Wind never varies, no night ventilation, no increase in ventilation in summer.	See Figure 37: as n increases, lightweight construction becomes more favourable. No night ventilation is a significant assumption as this is one of the effects that make thermal mass work. Night ventilation should perhaps have been included in the model but the author had to stop somewhere.
The thermal mass is isothermal, i.e. admittance is idealistically high.	In reality, a decrement factor should be included. The penetration of heat decreases with distance. Therefore, perhaps the 'intentional thermal mass' region of all the graphs (the pink region) should be slightly shifted to the left.
The occupants are perfectly conscientious and always program the building control system correctly; their schedule is always fixed.	In reality there would be times when the heating/cooling is on and the occupants are not in. This would affect all thermal masses.
The only U-value losses which occur are through the walls – floors and ceilings are considered as perfectly insulated!	Heat loss coefficient should be higher – see Figure 37 for the effect – which probably tends towards favouring lightweight construction.

ASSUMPTION	PREDICTED EFFECT ON RESULTS
The comfort zone is always between 20°C and 28°C.	In fact, some simulations which have not been included in this thesis were carried out to discern the importance of the choice of comfort zone. It was found that the definition of the comfort zone has to change by quite a few degrees to have a non-negligible effect on the shape of the graph.
Embodied energy is not considered throughout	Thermal mass might look less favourable.
Precooling as well as preheating is available and desired in the dwelling	See Appendix 3.

There are many other assumptions; it is hoped that the most important ones have been mentioned either in Table 8 or elsewhere. However, there is no real way of knowing whether or not the most important effects have been captured without finding an exact physical replica of the hypothetical dwelling.

6.7. Conclusion to chapter 6: insight gained in this chapter

Here is a summary of the results from the final model; i.e. what it appears that we have learned. How much we trust them is saved for chapter 7.

- We have not proved that occupancy schedule is the most important factor, but we have observed (see Figure 37) that it is important enough to impact what other graphs look like. In other words, it is important enough to show explicitly in other graphs: one cannot take an informative 2D slice through the model space without showing which occupancy schedule is being referred to.
- We have seen that both increase in and shifting of occupied hours contribute to the above.
- We have observed that predictability of energy use can be more difficult at low thermal mass.

We have also seen that:

- Sometimes incrementing thermal mass causes little change in energy use, and sometimes great change, due to the non-smooth nature of the objective function.
- Changing internal gains and size of building does not change the shape of the energy use versus thermal mass graph.
- Clutter makes a big difference.
- Splitting the curve into the heating component and the cooling component is helpful, and that the resultant curve can shift from heating-dominated to cooling-dominated and vice versa.

7. WHAT CAN BE LEARNED

This section, which will review the journey taken over the whole thesis, is more broad than the conclusion to chapter 6, which was just about the results from the final model.

7.1. Lessons for a modeller

(The author counts her future self here)

This thesis might seem a roundabout way of getting to a final model, but its development from a philosophy to a simple equation to a more complicated program has facilitated understanding of the system on the way: for example, we had the ability to solve simultaneous equations analytically and rearrange to whatever term suits us, the ability to test the effect of 'clutter' very simply, and the ability to test effects the sizes of which are unknown to start with (such as wall-air interaction) and know what is happening in the results. However, the objective function stops being continuous and therefore differentiable/integrable analytically as soon as slightly realistic inputs are included.

It follows that much more testing needs to be done before deciding what is important to include, since there does not seem to be a collective opinion on this. Hard work that results in a decision to omit an effect (such as in Appendix 2) makes the model simpler, and therefore later work easier and more insightful.

It is obviously advantageous to be able to create the kind of curve which can be split into heating and cooling components, and whose shape can therefore be explained by features of these curves respectively (e.g. to see whether it is pretempering or maintaining energy that is giving the curve a shape at its particular thermal mass). However complicated the program, the author would not want to lose this feature of how the results are obtained.

Continuous-spectrum analysis of thermal mass seems more useful than scenario analysis as is found in most of the literature, as this shows trends

better than finding several sparse points in the model space. Analysis can be done on this objective function, for example to discern the reason for any discontinuities present.

However, at the end of the day, this model is fairly far-removed from empirical data – this fact must not be forgotten.

7.2. Lessons for an architect

This study began by acknowledging that the answer to whether thermal mass should be used in a building is, “It depends”. In this study, we have proposed that the most important factors in “It depends” are: occupancy schedule, then heat loss parameter, then building size and internal gains. We have also seen that the contents of a room add significant thermal mass to it, just at a point where that mass *matters*, so if reliability in terms of energy performance is what is desired, building deliberately heavyweight is probably a good idea.

“It depends” means that the model space is not simple. It would be excellent if architects could gain insight into why, and which factors become important with which other factors. In Appendix 1, a preliminary attempt is made to design a tool to allow people (not just architects, but anybody) to explore the model space themselves.

7.3. Lessons for those who like optimizing

At the start of conducting this study, the author thought that she would write a fairly complex model and link it to an optimizing algorithm to find how to use thermal mass in dwellings. She now thinks that this is unsuitable for the following reasons:

- Having seen the benefit of the *user* doing the optimization by exploring the model space, it seems more suitable to think about how to explore a model space oneself (See Appendix 1).

- More specifically: consider Figure 20 – the first graph of the objective function produced. If an automated optimization procedure were used, there is a chance that it would incorrectly return the local minimum (see e.g. Coley & Schukat, 2002) after the 'kink' region. Now, if the building were specified to be constructed using this amount of thermal mass, and if it were incorrectly constructed to have just a little less thermal mass/clutter, its energy use could dramatically increase. There is a need for the designer to understand that this might happen – this is more likely if the designer has a feel for the shape of the objective function, which is again more likely if automated optimization is not used.

It is extremely understandable that big models whose objective functions take hours to calculate are better off using optimization algorithms which search the model space efficiently, but the point is that the model should not be made unnecessarily big to start with, and that there should be more literature to help building modellers cut down on less important elements.

7.4. Reflection on the specific application of the general discussion on modelling philosophy in chapter 2

The model-based science described in Chapter 2 gave us instructions to capture the essence of the system, to validate/verify our model appropriately, and to see the results through an appropriately cautious lens. Unfortunately, we do not know if we did capture the essence of the system since we cannot properly validate it, but at least we have carried out the cautious results-viewing!

As was also shown in Chapter 2, there seems to be a lack of literature on rules for modelling buildings, so we followed a general method of building up an equation, testing parts of it along the way, and solving it for many points in the model space.

However, even if the results could be validated/verified properly, there is an interesting limit to the accuracy of the model anyway. Chapters 3 to 5 try to

gradually improve the ‘accuracy’ of the equation; however, when occupancy is involved, this changes. The issue is not how to approximate something knowable – it is simply that a designer probably *cannot know* the future occupancy schedule of a dwelling in the design stage. This is reminiscent of something like the Uncertainty Principle in which there is a fundamental limit to our level of knowledge which is not our fault.

But philosophical comparisons aside, this limit of knowledge brings up an interesting question: what should be done in this situation? Do we design for the ‘most likely’ occupancy scenario, or design for the most likely *range* of situations, or design different houses and advertise them for different types of occupant?

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8. CONCLUSION

8.1. Summary

In this study, a model of the use of thermal mass in dwellings was developed over a number of iterations. In each one, experimental simulations were carried out to learn about the system. The final model enables overall annual energy use, which is a sum of heating and cooling energy use, to be calculated over a range of thermal masses in a space, and for other parameters to be varied. The model space was explored and a number of features observed, including the location of important regions.

Some predictable and some surprising results were found. An example of the former is that at high insulation, high convective losses render thermal mass a hindrance. An example of the latter is the high sensitivity of the objective function in the thermal mass region where contents of the room lie in a lightweight construction. Also indentified were the critical performance parameters. It was found that occupancy schedule (both the time of day that occupants are in, and the length of the period in which they are out) is important, as well as the heat loss performance of the dwelling.

8.2. Further work

There was some work done here to understand what in each case produced what part of the curve, which is a good start. However, a more thorough analysis would be better, and is already possible, given that the program structure allows breakdown of energy use due to heating and cooling (see Appendix 4). Perhaps annual energy use could be split up into seasons – one piece of analysis which was not attempted in this study is the calculation of utilisation factors in different seasons – see for example Figure 3 of ISO 13790 (2004).

Model space visualisation has been brought up and by no means resolved. As is discussed in Appendix 1, experiments would need to be carried out to find

the most effective means of communicating the shape of the model space to those who might take advantage of this knowledge.

8.3. Concluding remarks

This thesis is both about the process of developing a thermal mass model and the results obtained from it. Therefore it is appropriate to conclude on both subjects.

It is fitting that the whole problem, which is about trade-off between the benefits and drawbacks of thermal mass, has ingrained in its solution method a trade-off between solution time and real-world likeness. We have seen how the inclusion of more factors firstly caused us to rely on a numerical solver instead of analytic insight, then started to take a long time, then caused model space visualisation to be difficult. It is hoped that the latter obstacle is seen as an opportunity to think about how to communicate a non-trivial solution to anybody who wants to know about it.

The literature did not conclusively warn that occupancy would be the most important factor in the debate about when to use thermal mass in dwellings, but perhaps this should have been guessed. Occupant behaviour is a big ‘black box’ area and scientists sometimes like to avoid it. However, it is essential to understand how people relate to buildings, so that they can be built correctly and operated by occupants efficiently. For each dwelling designed, it would be ideal to know what kind of occupant will be in there, but without such knowledge, perhaps the most reliable solution is to use thermal mass.

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APPENDICES

Appendix 1: Enabling exploration of the model space

In this section, we consider enabling people to develop intuition into the way energy use over a range of thermal masses changes over variation in circumstances by the example of a Graphical User Interface (GUI).

Intuition needs to be developed by building scientists, engineers, architects, and people who use buildings – in fact, the more people who want to learn about thermal mass, the better.

A proper investigation into how different types of people like to learn, and hence the best way to translate insight gained from a graph in this study into that which makes sense to others, would require a lot of research. This field is called 'User-centred design', and requires first of all a systematic and structured collection of users' requirements, then an iterative design process in which the user is involved at every stage, then usability tests (Courage & Baxter, 2005). None of these is attempted here; instead, the author will draw from a couple of examples in the literature. Therefore, this section is not included in the main text but instead as an appendix; it is not meant to be a completed piece of work but rather to provoke thought about model space exploration by those who have not necessarily worked on the model.

Case studies

The two case studies to be used are the 'simple design tool for the thermal study of dwellings' in Gratia & De Herde (2002) and 'Jacaranda', a chemical processing design tool created by Eric Fraga (see e.g. Fraga et al., 2000). This quote from the documentation from the latter gives the motivation for using visualisation:

"*The key difficulty in using automated systems in process design is the complexity of the underlying problem. Especially in chemical engineering, the interactions between different elements in a process may be not only complex*

but not well understood. The use of automation can lull the engineer into believing that the results obtained by the solution approach is the best possible, without questioning the underlying models which lead to this solution. Visualisation, often with data analysis, can be useful in providing the engineer with extra information about the quality of the solution obtained and how it relates to the actual design space." (Fraga, E., unknown date.)

This quote, although referencing chemical engineering, relates to the thermal mass problem in the sense that unless people are presented with the whole model space, they lose insight as to the robustness of the solution (amount of thermal mass) they choose.

The resulting program made by Fraga was a Java tool, of which an example of the GUI is shown in Figure 38.

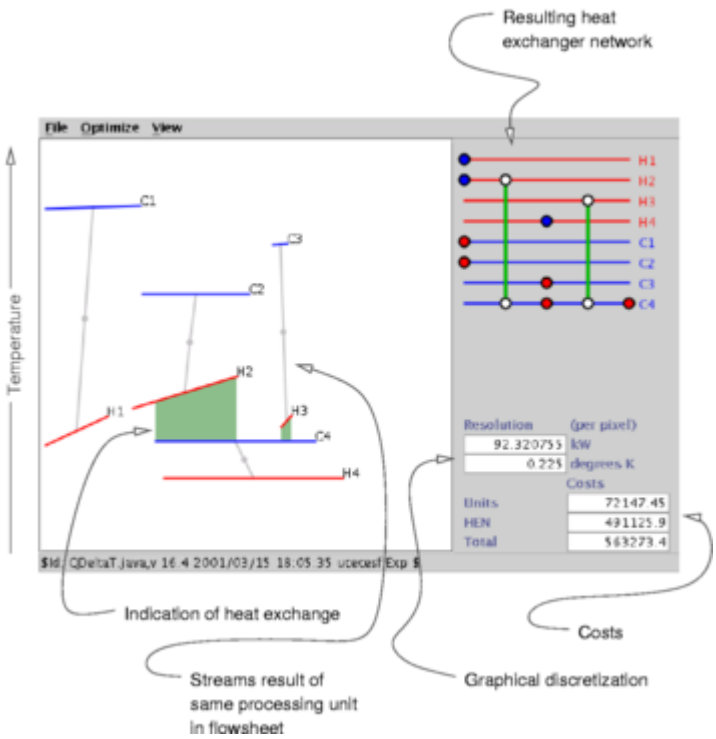

Figure 38: An example of a screen from Fraga's tool.

The user can play with different configurations of a process, changing one variable at a time and watching the effect on cost or another objective function. This is preferred by Fraga to a tool which optimises the process automatically, and for example may cause the user to lose insight as to which factors could cause the minimum not to be obtained.

In the case of Gratia and de Herde, a comparison was made between the running time requirements and amount of data input of a proper dynamic thermal simulation to those of a tool suitable for architects to use in real time to design buildings. It was concluded that a way to combine the two is to run many parametric analyses on a dynamic model, store the results, and design the user tool to read from these pre-obtained results. This seems an extremely suitable way of proceeding in the case of the thermal mass problem, so this methodology will be used. An example of Gratia and de Herde's user interface is shown in Figure 39 (in this particular case they are showing the user that compactness of a space is a big influencing factor on energy use):

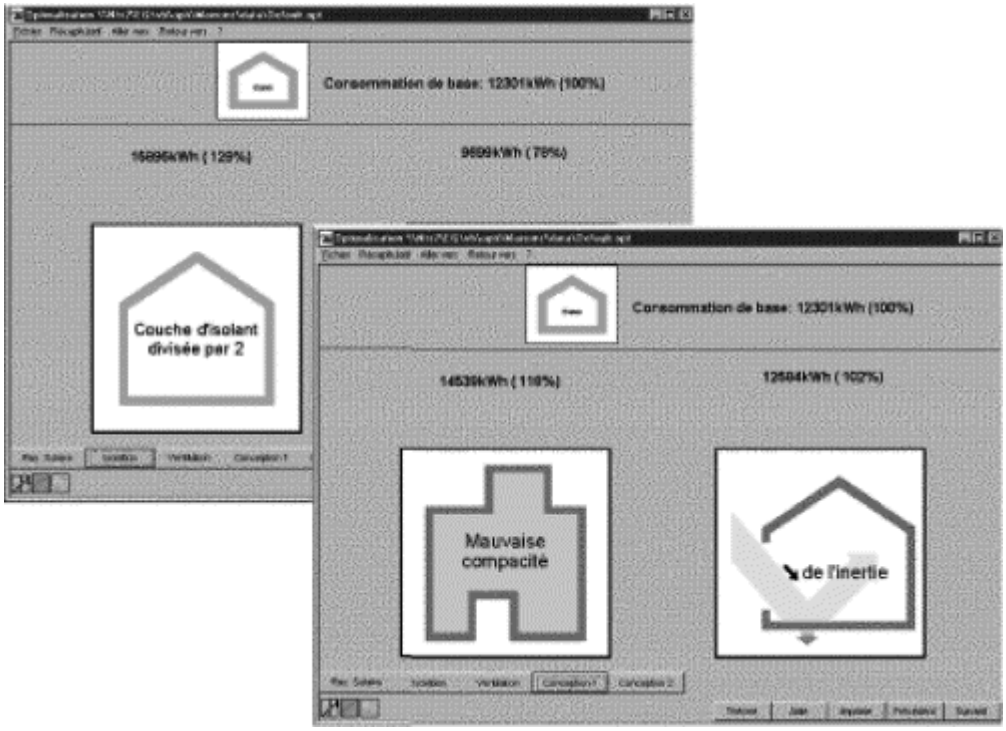

Figure 39: An example user interface from Gratia & de Herde.

From these two case studies, the philosophy and approximate program structure of the Fraga tool and the methodology of the Gratia & de Herde tool are made use of here.

Making a thermal mass tool

It was decided that, like Jacaranda, the tool would be written in the Java programming language, since this language can display graphics effectively, read from a data file, is platform-independent and can therefore run on any machine, and is object-oriented and hence its functionality easily extended if desired.

A screenshot from the tool is shown in Figure 40, for reference during the subsequent explanation. Further screenshots are shown in Figures 42-44.

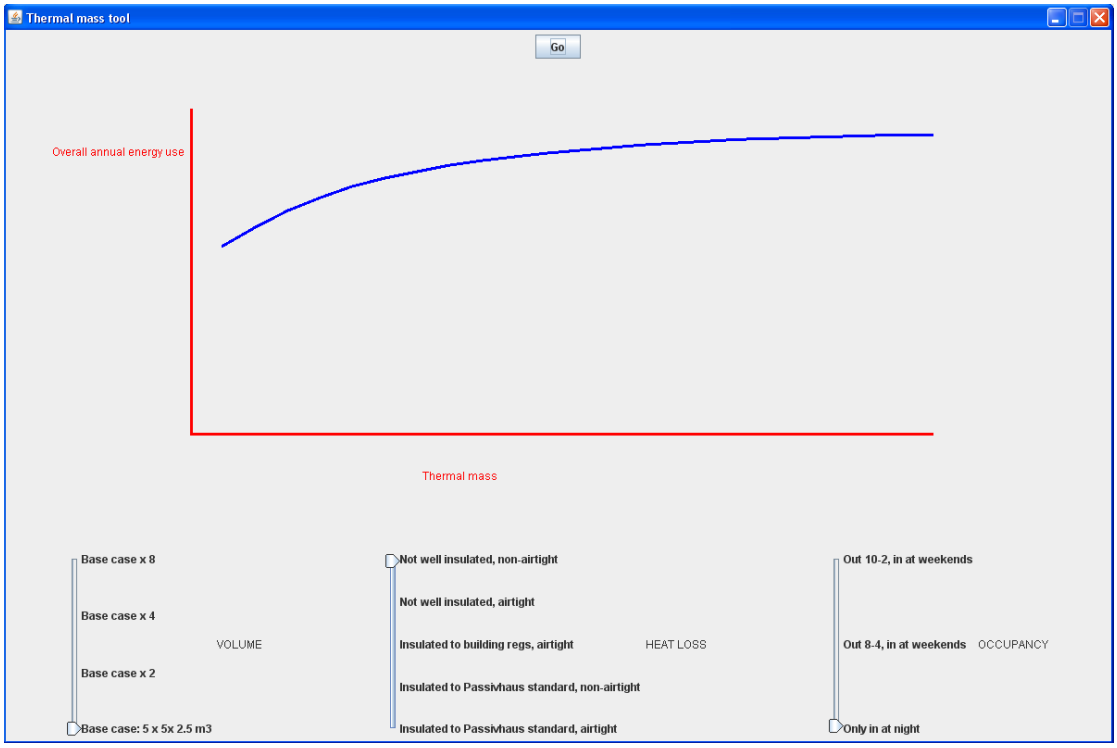

Figure 40: Screenshot from the Java tool.

What the user can change, and how

In Figure 40 are three sliders. The user can change the dwelling volume, the heat loss characteristics and the occupancy schedule. The graph is supposed to change when the user moves the sliders, so that the effect of changing one variable at once can be seen. However, the author is not competent enough at programming to make the graph update as the slider is moved, so a button

named “Go” was included, and is pressed by the user when he/she has set the sliders in the desired position.

Please note that in fact the ‘Volume’ slider is at the moment for illustration purposes only and does not work, as even in the parametric simulations, the whole model space was not explored. The author also could not find out the correct Java to put the clutter and intentional thermal mass regions on the graph.

Retrieving the data

In chapter 6, we discussed computer visualisation of the model space; that is, how to create a data table with all the results in over many changing parameters. This is an important question here, since Java has to read from the results file so it needs to be structured in a suitable way.

It was decided that Microsoft Access would be used to store the results of the simulations, and then a query run to extract the correct row of data.

The data is stored as shown in the extract of the database in Figure 41, so that a query such as the following example will extract a row - the results of energy use across all thermal masses for one particular combination of inputs:

```
select * from data where Volume="+vol+ " and HeatLoss="+heatLoss+" and OccupancySchedule="+occupancy
```

(where vol, heatLoss and occupancy are the readings from the sliders that the user has changed.)

One row is selected by the query

data

Volume	HeatLoss	OccupancySchedule	A	B	C
1	1	1	682775077.894476	774305127.189134	820106536.597949
1	1	2	998672976.047853	1040988452.64995	1056163936.12708
1	1	3	1012188588.58939	1006829590.17429	957723499.56462
1	2	1	2122580870.07078	2321632382.30864	2477017411.81246
1	2	2	2916418382.46048	2975374843.60828	3019501255.32002
1	2	3	3184106072.89291	3147944389.31411	3124222075.9157
1	3	1	883093211.470328	999616684.641484	1065453521.19723
1	3	2	1288085450.12388	1335486059.72208	1364689546.11703
1	3	3	1316355712.65296	1325679571.01643	1303627816.94997
1	4	1	2052913478.98865	2247235514.29563	2395951987.65999
1	4	2	2829921024.05094	2883424779.67039	2919949515.39287
1	4	3	3082891386.37417	3024243189.99863	3017407639.59056
1	5	1	2927278961.42986	3150458879.83284	3356316593.63475
1	5	2	4015746226.9148	4137120034.6869	4204907841.66021
1	5	3	4513290220.52356	4457224538.97395	4406845662.06374

Figure 41: An extract from the Access data file

How the data is displayed

The data is plotted on a two-dimensional graph. It could be argued that this format is not a very interesting way to show the model space, and that the program was obviously written by a physicist. One benefit of displaying a graph is that regions of sensitivity can be observed. If the research described earlier of user preferences etc were to be carried out and it were found that a graph is not the right medium to convey the results, one thing to think hard about would be how to get across the idea of sensitive regions on a graph without necessarily presenting one.

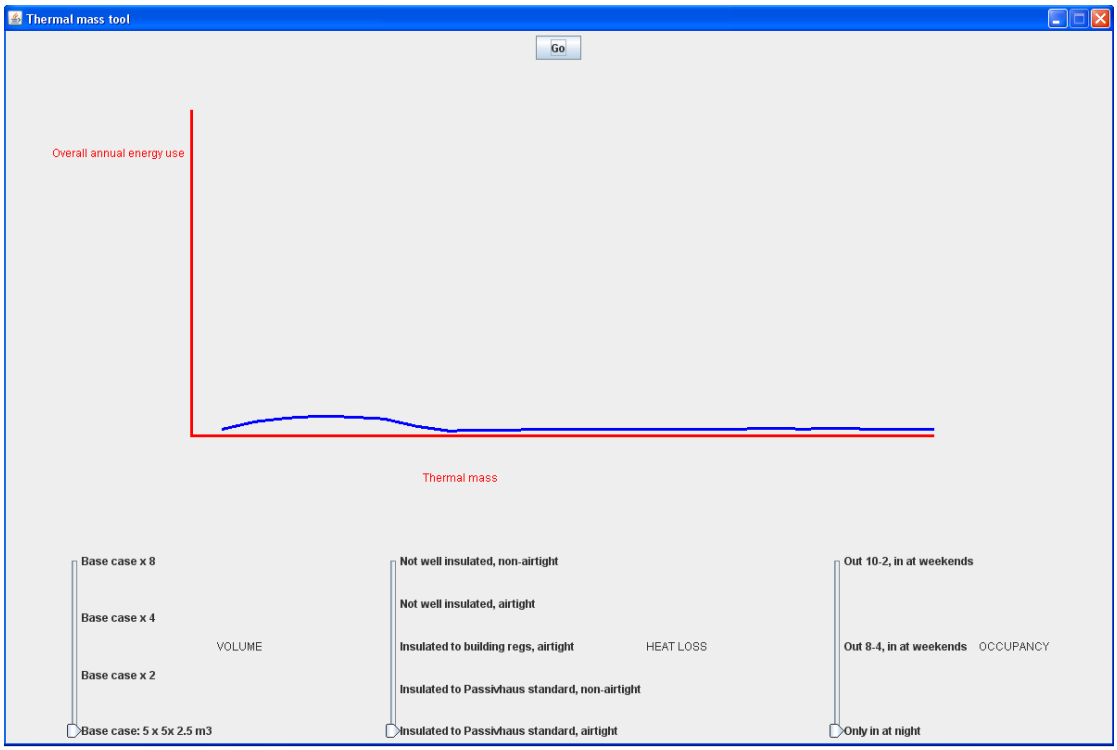

Figure 42: The "Control Experiment" from Chapter 6.

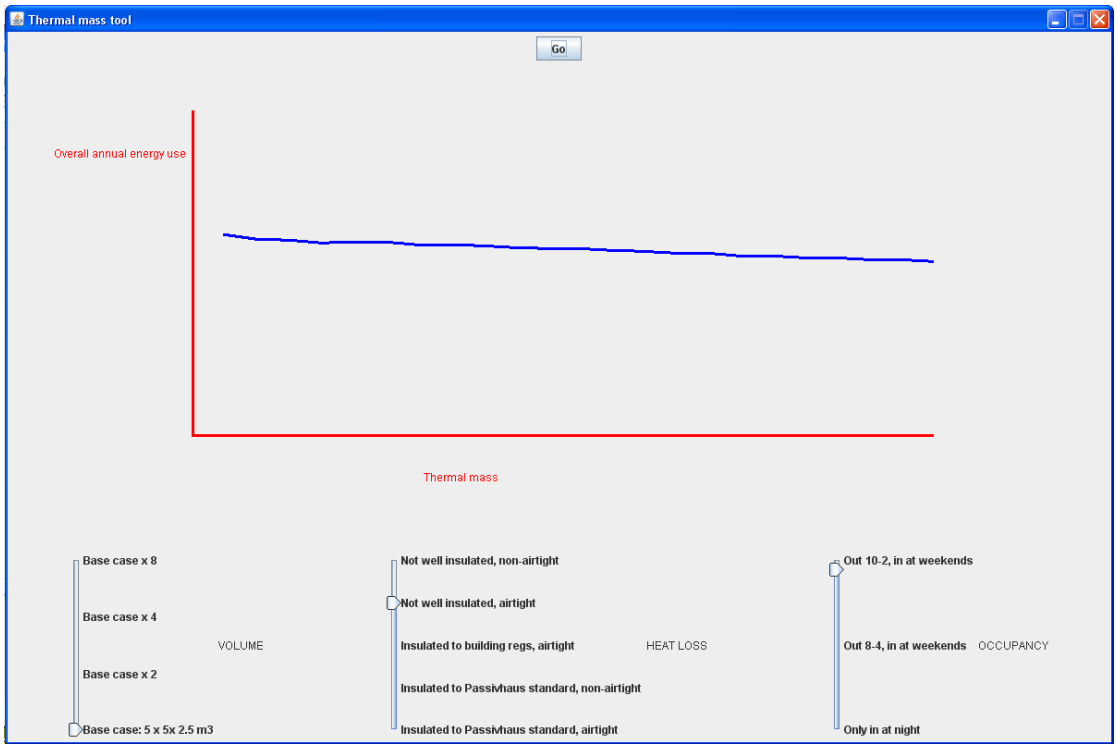

Figure 43: A further screenshot.

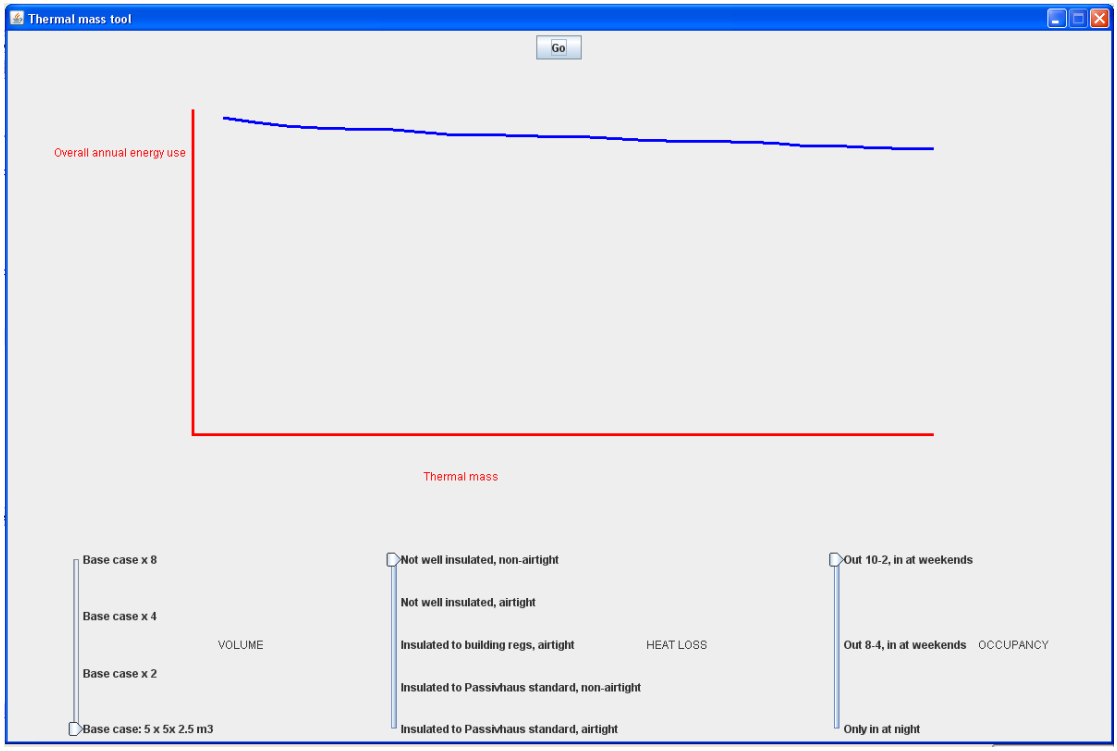

Figure 44: A further screenshot.

Conclusion

In conclusion, even though this tool is not well researched, it is an initial attempt to make the model space more user-friendly, and challenges us to think about how to do this.

The tool is included in the electronic version of this thesis.

Appendix 2: Whether to treat walls and air as separate nodes

Is heat transfer between walls and air an important effect when modelling thermal mass? A simulation experiment will now be carried out to discern whether this effect makes a significant difference to the temperature over time. If so, the final model should include two coupled differential equations – one representing the air and one the walls. If not, one equation representing the whole system should suffice.

One observation which makes the wall and air equations differ is that gains and losses are now particular to either the wall or the air. In this experiment, solar gain warms up the wall, and gain from radiators and occupants warms up the air. In reality this is not at all so clear–cut, one reason being the phenomenon of 'primary' and 'secondary' thermal mass, but this is beyond the scope of this thesis. Also please note that the floor is ignored in this treatment.

One-node equation

$$G(t) = C\frac{dT}{dt} + H(T - To(t))$$ Equation 15

If this were decoupled into separate equations for walls and air,

Wall equation:

$$Cw\frac{dTw}{dt} = hA(T - Tw) + Gs$$ Equation16

Air equation:

$$Ca\frac{dT}{dt} = -hA(T - Tw) + Gns - H(T - To(t))$$ Equation 17

Where:
C_w = thermal mass of wall
T_w = wall temperature
T = internal air temperature
G_s = solar gain
C_a = thermal mass of air (clutter is neglected in this experiment)

G_{ns} = all gains that are non-solar, therefore first go into the air not the wall.

Let us be a bit more specific and define G_s and G_{ns} as constants, and $T_o(t)$ as $T_o\sin(wt)$, i.e. a basic sine function to describe external temperature

Equations 16 and 17 are coupled first order differential equations and can be solved analytically. However, the analytic solution is extremely complicated, so we will solve them in Matlab. We will then compare the result to the solution of Equation 15.

Two cases were tested: summer and winter, day and week. Table 9 shows inputs to the experiment:

Table 9: Inputs to Equations 15, 16 and 17.

Input	Summer value	Winter value
C, J/K	1575900	1575900
C_w, J/K	1500000 (corresponding to 50mm of thermal mass)	1500000
C_a, J/K	75900 (the thermal mass of all the air in the space)	75900
G_{ns}, W	0	1000 (corresponding to having the heating on)
H, W/K	20.1	20.1
Initial condition: T at t = 0, °C		0
Initial condition: T_w at t = 0, °C		-5
T_o, °C	10	10

K (constant added to T_o to differentiate between summer and winter)	20	5

Results

One week in summer and one week in winter, with the single- and coupled-equation systems, are shown in Figures 45, 46, 47 and 48:

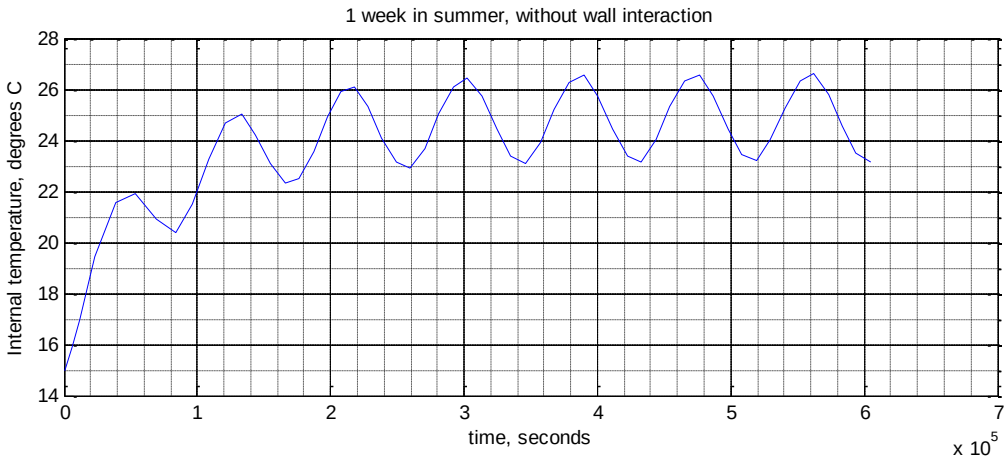

Figure 45: Temperature profile over summer week, no wall interaction.

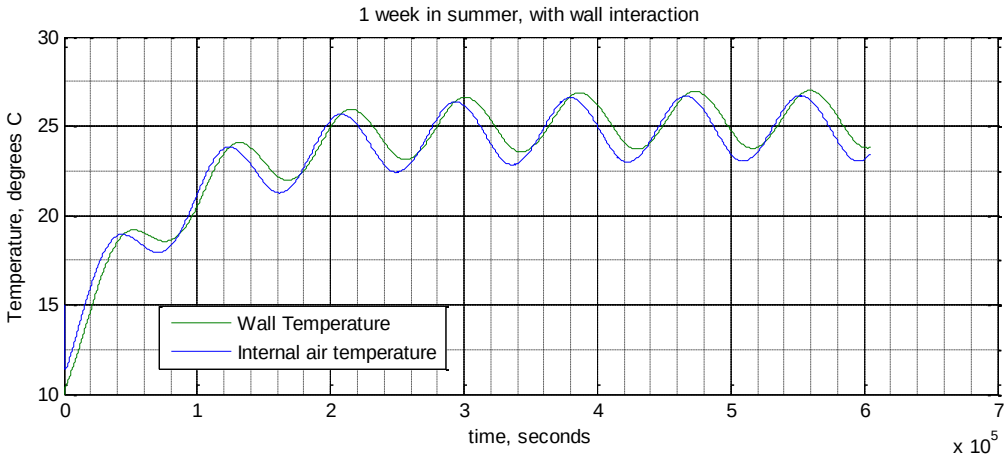

Figure 46:Temperature profile over summer week, with wall interaction.

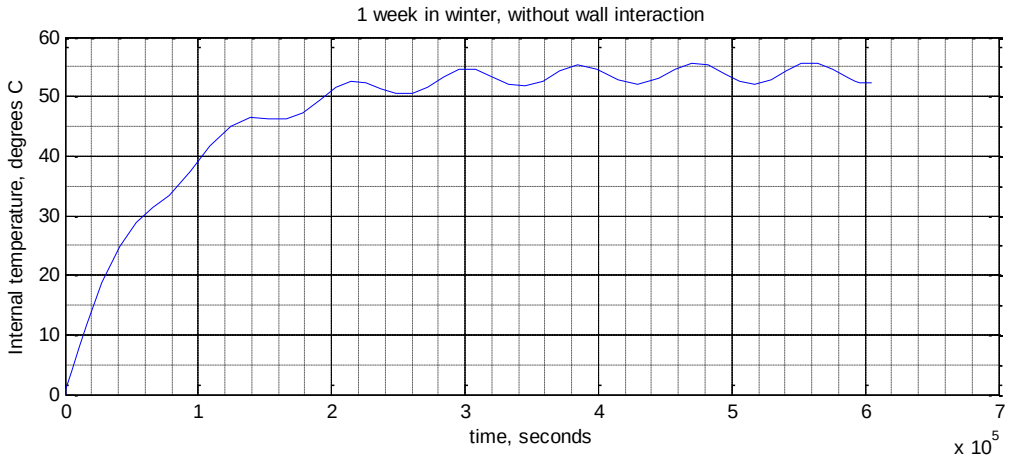

Figure 47:Temperature profile over a winter week, no wall interaction.

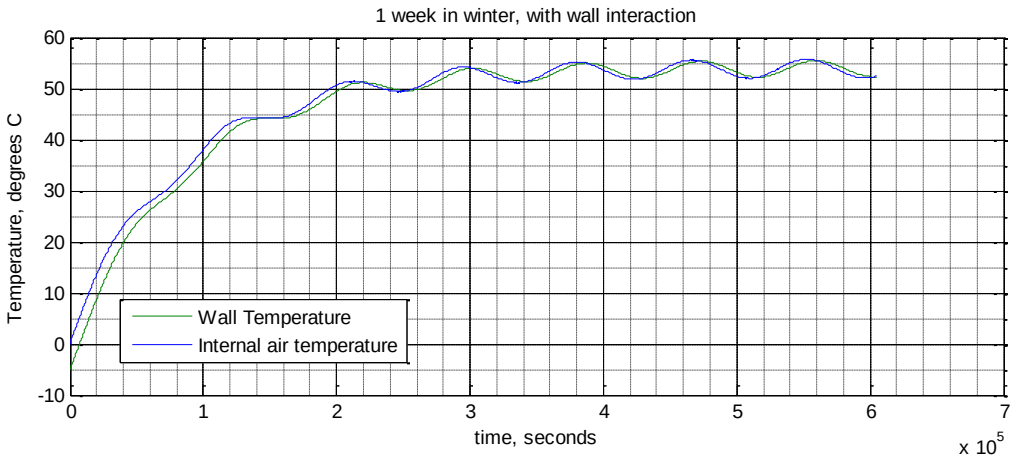

Figure 48:Temperature profile over a winter week, with wall interaction.

Note that it is not the *absolute* temperatures reached that we are concerned about, since some assumptions have gone into this model that will not be part of the final equation (for example, the reason the temperature goes so high in winter is the constant solar and other gains!). It is actually the difference between the times of peaks and troughs, and the difference in temperatures at any point, between the cases, which interests us.

It seems from these graphs that the phase difference between the behaviour of the one-equation system and any component of the two-equation system is a matter of minutes. As long as the time-step in the final model is an hour or

more, this phase difference will not matter. Also, the temperature difference between the air and the walls is a couple of degrees at most; usually much less. This might sound like a fairly significant effect, but compared to the other sources of error in the model (for example, solar gain being between the hours of 6 a.m. and 6 p.m. every day regardless of the season) it is negligible.

Therefore, it seems sensible not to use the twin-equation system, since it does not make that much difference from the one-equation combined system.

Appendix 3: Multi-criterion optimization

A presumably significant assumption made in the final model developed in chapters 4 and 5 is the availability of domestic cooling, to the extent that people use pre-cooling in their dwellings. Cooling is not the norm in UK dwellings at the time of writing; whether it will become so in the future is not known. If a house has heating but not cooling, the temperature will presumably increase above the comfort zone at some points in the year. Thermal mass will help mitigate this effect, but could be a hindrance in the winter as preheating times increase.

Thus, the problem becomes a trade-off between winter energy use and summer overheating, these two phenomena arguably not being expressible in the same unit. When this occurs, a decision on how much thermal mass to use becomes a two-criterion optimization. In fact, this is probably the norm: “Since designers rarely consider only one criterion in the decision-making process, multi-objective optimization models have been proposed.” (Wang et al., 2005).

How to deal with this in general and in this thermal mass problem will now be discussed, along with benefits and drawbacks of this treatment.

Methods

Multi-criterion decision making (MCDM) can be performed in several ways (Van Veldhuizen & Lamont, 2000). In 'a priori' MCDM, multiple objective functions are combined into one by weighting each one and minimising a weighted sum. An example given in Wright et al. (2002) is that a designer might decide that capital cost is twice as important as operational cost of a building. Optimization is then carried out. However, this approach produces only a single result, which does not bring much insight into anything, not least how the different objective functions interact.

The opposite of this method is 'a posteri' MCDM, in which optimization occurs over a range of different weightings of objective functions, and a range of optimum solutions is presented as a 'pareto front'. Then the designer chooses which solution is most suitable. In this way, the trade-off between objective functions can be seen, another of Wright's examples being how much operational cost can be reduced if capital cost is increased just a bit. A pareto front looks like the example in Figure 49, sourced from Wang et al. (2005), whose objective functions are life cycle cost and life cycle environmental impact. The black triangles show the 'front' of optimum solutions found throughout the running of a genetic algorithm.

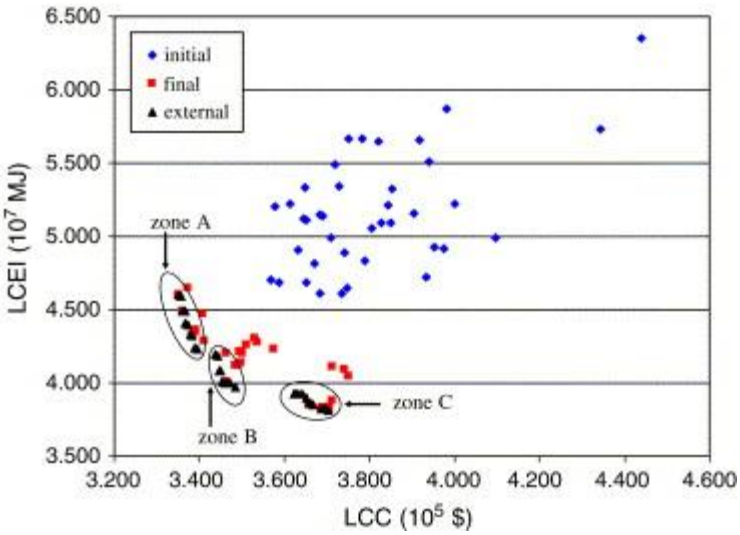

Figure 49: Figure 3 from Wang et al. (2005).

The a posteri approach seems to be that most commonly adopted in the literature.

Drawbacks

i. Optimization is usually carried out without sensitivity analysis.

The illustration of trade-off in Figure 49 is just scratching the surface of all the sensitivity analysis which needs to be carried out to understand the model space. In the thermal mass problem, if thermal comfort and energy use were the two objective functions, the equivalent of Figure 49 would be a pareto front showing trade-off between one objective function and the other – i.e. both dependent variables. However, this would not show anything about all the effect of the independent variables – the rest of the model space. In all the literature on multi-objective optimization in building simulation, the author can only find one example of work taking slices of the model space which show sensitivities of an objective function with respect to an independent or semi-independent variable – again this is in Wright et al.(2002). In this paper it is shown that, for the hypothetical building being analysed, temperature setpoint does not affect energy use at one time of day (10 a.m.) but does later on (4 p.m.). This result highlights the complexity of the model space and the need for this type of sensitivity analysis!

ii. The unrelated nature of the solutions.

Another drawback of the pareto curve method is the discrete nature of points on it. A curve drawn through pareto optima may look continous (or may not), but the individual solutions of which it is formed are each a result of a set of input variables, whose values from one solution to the next may be completely unrelated. This is demonstrated by the pareto front shown in Figure 50, sourced from Caldras & Norford (2003), in which building envelope features are the independent variables and heating and lighting enery are the two objective functions.

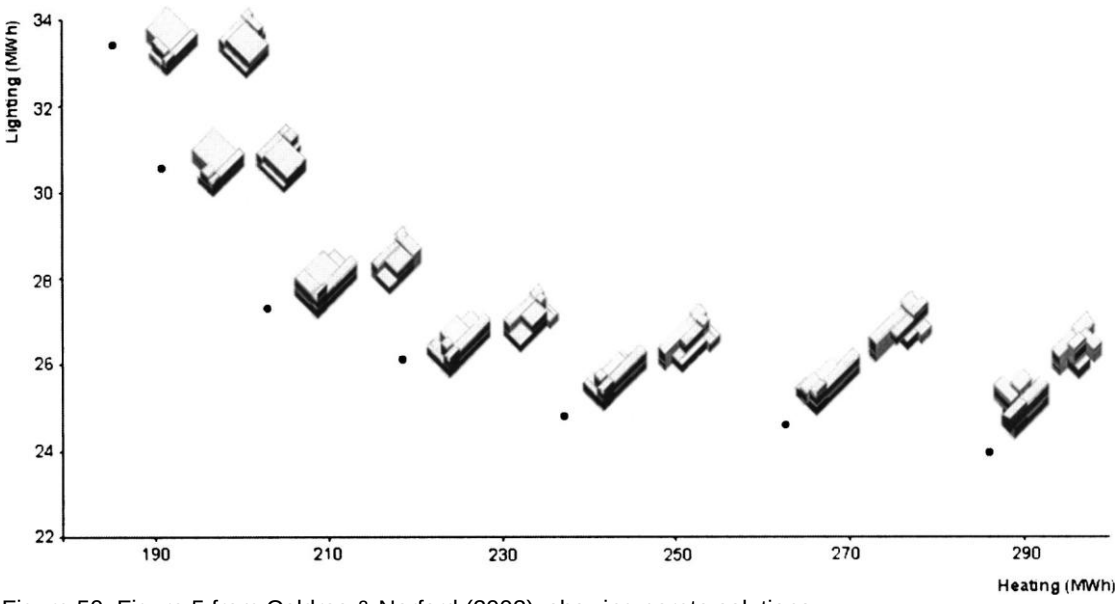

Figure 50: Figure 5 from Caldras & Norford (2003), showing pareto solutions.

Each solution on the pareto front is a different building form, not necessarily related to those either side of it. Therefore there is no insight here into what happens if the real building is constructed slightly differently, e.g. the windows are too small. (This represents an example of local/quasi-local sensitivity – changing one/a few variable(s) whilst keeping the rest constant.) In other words, it does not look like the designer can *explore* and *learn about* the model space, just *look at* and *hope to replicate the best bits of* it.

Conclusion

In conclusion, if two objective functions of, e.g., heating energy use and thermal comfort, were used in the thermal mass problem, there is no doubt that the problem would become more complex regardless of which treatment were used. Unless weightings for each objective function could easily be decided upon (this is not the case as people do not ascribe the same boundaries or value to thermal comfort), it might be appropriate to use a multi-criterion optimization algorithm, since just looking at the model space would take a long time given that it would have to be done for different weightings of the objective functions. A posteri decision making from the set of solutions given for different weightings could be carried out. However, we have noted the loss of insight from this method unless accompanied by rigorous

sensitivity analysis. As in all the way through this thesis, an increase in complexity of the solution method would lead to a loss in insight.

Appendix 4: The Matlab code for the final program

```
/* Here is the code for the main function of the program which calculates overall annual energy use. This example shows thermal mass and heat loss parameter (H) being varied. The occupancy scenario in this example is number 2, i.e. the dwelling is occupied all weekend, and between 4pm and 8 am on weekdays. (Hence, the pretempering comes on at 3pm.)

*/



function OverallEnergy = MainUberWkend()


maxk = 6;

maxj = 8760;

maxi = 23;

Q = zeros(length(maxi),length(maxj));

Hvec = [15.41667 17.91667 45.83333 20.625 25.83333 36.25];

global k;

global i;

global j;
```

```
T = zeros(length(maxi),length(maxj),length(maxk));

global T_init;

T_init = 20;

global G;

G = zeros(maxi, maxj, maxk);

HeatingMatrix = zeros(maxi, maxj, maxk);

CoolingMatrix = zeros(maxi, maxj, maxk);

OverallHeatingEnergy = zeros(maxi, maxk);

OverallCoolingEnergy = zeros(maxi, maxk);

OverallEnergy = zeros(maxi,maxk);

global InternalGainsFactor;

InternalGainsFactor = 1.5;



for k = 1:1:maxk

    H = 3600*Hvec(k);



for i = 1:1:maxi


C =    (500000-250000) + i*250000;
```

```
for j = 1:1:maxj

    type = rem(j, 24);

    t = j;

    tspan=[t-1 t];




    if rem(j,24) ~= 0

    d = 1 + j/24 - (rem(j,24)/24); % works almost every
hour

    else

        d = j/24; %works the rest of the time

    end




    if rem(d,6) == 0 || rem(d,7) ==0          % if it is
the weekend

         [A,         B]         =         ode45((@(t,Tfree)
(1/C)*InternalGainsFactor*100*3600*heaviside(sin(2*pi*(t+
6)/24))       +       (1/C)*250*3600*heaviside(sin(2*pi*(t-
6)/24)).*(sin(2*pi*(t-
6)/24).*sin(2*pi*(t+500)/(8760*2.23)))    -  H*T_init/C  +
(H/C)*       (5       +       16*sin(2*pi*t/(8760*2))       +
5*sin(2*pi*(t+500)/(8760*2.23)).*sin(2*pi*(t+15)/24))),ts
pan,[T_init]);
```

```
            T(i,j,k) = B(length(B),1);


        Q(i,j,k) = T_init - T(i,j,k);


        if  T(i,j,k) > 20 && T(i,j,k) < 28
            G(i,j,k) = 0;
        else
            G(i,j,k) = (1/3600)*C*Q(i,j,k) ;
        end


        if T(i,j,k) < 20
            T_init = 20;
        elseif T(i,j,k) > 28
            T_init = 28;
        end




   else                                    %if it is not the
weekend


    if type <=8
```

```
[A, B] = ode45((@(t,Tfree)
(1/C)*InternalGainsFactor*100*3600*heaviside(sin(2*pi*(t+
6)/24)) + (1/C)*250*3600*heaviside(sin(2*pi*(t-
6)/24)).*(sin(2*pi*(t-
6)/24).*sin(2*pi*(t+500)/(8760*2.23))) - H*T_init/C +
(H/C)* (5 + 16*sin(2*pi*t/(8760*2)) +
5*sin(2*pi*(t+500)/(8760*2.23)).*sin(2*pi*(t+15)/24))),ts
pan,[T_init]);



        T(i,j,k) = B(length(B),1);


        Q(i,j,k) = T_init - T(i,j,k);


        if  T(i,j,k) > 20 && T(i,j,k) < 28

            G(i,j,k) = 0;

        else

            G(i,j,k) = (1/3600)*C*Q(i,j,k) ;


        end


    if T(i,j,k) < 20

            T_init = 20;

        elseif T(i,j,k) > 28
```

```
            T_init = 28;

        end


    elseif type <= 15

        [A, B] = ode45((@(t,Tfree)
(1/C)*InternalGainsFactor*100*3600*heaviside(sin(2*pi*(t+
6)/24)) + (1/C)*250*3600*heaviside(sin(2*pi*(t-
6)/24)).*(sin(2*pi*(t-
6)/24).*sin(2*pi*(t+500)/(8760*2.23))) - H*T_init/C +
(H/C)* (5 + 16*sin(2*pi*t/(8760*2)) +
5*sin(2*pi*(t+500)/(8760*2.23)).*sin(2*pi*(t+15)/24))),ts
pan,[T_init]);

        T(i,j,k) = B(length(B),1);

        T_init = B(length(B),1);

        G(i,j,k) = 0;


    elseif type == 16 % this is the pretempering hour
```

```
      GTesterUber(H, C);


      if  G(i,j,k) > 0

        T_init = 20;

      elseif G(i,j,k) < 0

        T_init = 28;









       end


       T(i,j,k) = T_init;

    else


        [A,          B]          =          ode45((@(t,Tfree)
(1/C)*InternalGainsFactor*100*3600*heaviside(sin(2*pi*(t+
6)/24))       +       (1/C)*250*3600*heaviside(sin(2*pi*(t-
6)/24)).*(sin(2*pi*(t-
6)/24).*sin(2*pi*(t+500)/(8760*2.23)))     -  H*T_init/C  +
(H/C)*       (5       +       16*sin(2*pi*t/(8760*2))       +
5*sin(2*pi*(t+500)/(8760*2.23)).*sin(2*pi*(t+15)/24))),ts
pan,[T_init]);
```

```
        T(i,j,k) = B(length(B),1);

         Q(i,j,k) = T_init - T(i,j,k);

         if  T(i,j,k) > 20 && T(i,j,k) < 28
            G(i,j,k) = 0;
        else
            G(i,j,k) = (1/3600)*C*Q(i,j,k) ;

        end

       %update T_init
        if T(i,j,k) < 20
            T_init = 20;
        elseif T(i,j,k) > 28
            T_init = 28;
        end

    end
    end
```

```
  if G(i,j,k) > 0

       HeatingMatrix(i,j,k) = G(i,j,k);

  elseif G(i,j,k) <0

      CoolingMatrix(i,j,k) = G(i,j,k);


  end   % end of the 'if d is or isn't a whole number'
loop


end %end of the j loop




  OverallHeatingEnergy(i,k)             =             3600*
sum(HeatingMatrix(i,:,k));

  OverallCoolingEnergy(i,k)             =             3600*
sum(CoolingMatrix(i,:,k));

  OverallEnergy(i,k)    =    OverallHeatingEnergy(i,k)    +
abs(OverallCoolingEnergy(i,k));




end  %end of the i loop
```

```
end  %end of the k loop



%write the results to an Excel file

 xlswrite('outputofmatlab.xls',    [OverallHeatingEnergy],
'Overall_heating_energy', 'B2');

 xlswrite('outputofmatlab.xls',    [OverallCoolingEnergy],
'Overall_cooling_energy', 'B2');

 xlswrite('outputofmatlab.xls',           [OverallEnergy],
'Overall_energy', 'B2');


 warning off MATLAB:xlswrite:AddSheet;


end



/*

Here is the code for the algorithm which decides on the
power at which the pretempering works for an hour. It
makes an initial guess, then iterates it until the indoor
temperature ends up at 20 or 28, whichever is
appropriate.

*/
```

```
function [G] = GTesterUber( H, C)


global G;

global i;

global j;

global k;

global T_init;

global InternalGainsFactor;

tol = 0.1;

deltaG = 50*3600;




if T_init < 20

    G(i,j,k) = 1000*3600;

elseif T_init >= 29.7 %NOT just 28, as if it's just above
28, it cools down by itself, and then the heating comes
on!

    G(i,j,k) = -1000*3600;

else

    G(i,j,k) = 0;

    [T,Y]                 =                 ode45((@(t,T3)
(1/C)*InternalGainsFactor*100*3600*heaviside(sin(2*pi*(t+
6)/24))       +       (1/C)*250*3600*heaviside(sin(2*pi*(t-
6)/24)).*(sin(2*pi*(t-
```

```
6)/24).*sin(2*pi*(t+500)/(8760*2.23)))    -  H*T_init/C  +
(H/C)*      (5      +      16*sin(2*pi*t/(8760*2))      +
5*sin(2*pi*(t+500)/(8760*2.23)).*sin(2*pi*(t+15)/24))),[4
314:0.1: 4315],[T_init]);



    T_init = Y(length(Y), 1);


    return;


end


for m = 1:1:100


[T,Y]                    =                    ode45((@(t,T3)
(1/C)*InternalGainsFactor*100*3600*heaviside(sin(2*pi*(t+
6)/24))             +             G(i,j,k)/C             +
(1/C)*250*3600*heaviside(sin(2*pi*(t-
6)/24)).*(sin(2*pi*(t-
6)/24).*sin(2*pi*(t+500)/(8760*2.23)))    -  H*T_init/C  +
(H/C)*      (5      +      16*sin(2*pi*t/(8760*2))      +
5*sin(2*pi*(t+500)/(8760*2.23)).*sin(2*pi*(t+15)/24))),[1
8:0.1: 19],[T_init]);


Yfinal = Y(11,1);
```

```
if T_init < 20


   if abs(Yfinal-20) <= tol

         break;

    elseif(Yfinal-20)> tol

        G(i,j,k) = G(i,j,k) - deltaG;

    elseif(Yfinal-20)< (-tol)

        G(i,j,k) = G(i,j,k) + deltaG;

    end


elseif T_init >

    if abs(Yfinal-28) <= tol

        break;

    elseif(Yfinal-28)> tol

        G(i,j,k) = G(i,j,k) - deltaG;    %Make the cooling
even greater in magnitude

    elseif(Yfinal-28)< (-tol)

        G(i,j,k) = G(i,j,k) + deltaG;    %stop the cooling
being as powerful

    end


end
```

```
end

G(i,j,k) = G(i,j,k)/3600;

end
```